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Yau-Chuen Wong

The Topology of Uniform Convergence on Order-Bounded Sets

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Author
Yau-Chuen Wong
Department of Mathematics
United College
The Chinese University of Hong Kong
Shatin, N.T./Hong Kong

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C O N T E N T S

INTRODUCTION

In studying ordered topological vector spaces, particularly important roles are played by two intrinsic topologies: the order-bounded (or order) topology and the topology σ_S of uniform convergence on all order-bounded sets. The order-bound topology was studied independently by Schaefer [1] and Namioka [1] , while the topology σ_S was studied by Nakano [1] and Dieudonné in the special case of locally convex Riesz spaces, and by Peressini [3] in a fairly general setting (he used the notation $o(E, E')$). A remarkable theorem of Nakano [1] (asserting that, for topological Riesz spaces, topological completeness follows from certain order completeness assumption) is one of the deepest results in the theory of locally convex Riesz spaces; the author showed, in 1969, that σ_S is relevant for establishing a converse of Nakano's theorem. Therefore it is interesting to seek some necessary and sufficient condition for a given locally solid topology $\mathcal{P}$ on E to be $o(E, E')$. One of the purposes of these lecture notes is to give such characterizations by means of some special classes of continuous linear mappings, and another purpose is an attempt to provide a unifying treatment of nuclear spaces and the topology $o(E, E')$. The guiding concepts in this approach are those of absolutely summing mappings and cone-absolutely summing mappings. These concepts are studied only in the general setting here; for the speciality of such an account in the Banach lattices setting the reader is referred to the excellent book written by Schaefer [3] .

The first chapter is a brief discussion of duality problems for ordered vector spaces, and of the constructions of the topologies $\mathcal{P}_F$, $\mathcal{P}_D$ and $\mathcal{P}_S$ (respectively the locally o-convex topology, the locally decomposable topology and the locally solid topology associated with $\mathcal{P}$). The second chapter mainly deals with some useful classes of locally solid topologies on certain vector subspaces of E^A, and studies the corresponding dual structures. The final chapter is devoted to a study of cone-absolutely summing mappings, of cone-prenuclear mappings and of the topology $o(E, E')$.

Throughout these notes, (i, j, k) will denote the k-th proposition or theorem in Chapter i, Section j.

Parts of these notes were delivered at Yale University, during the period from September 1973 to March 1974, and at McMaster University in Canada, during the period of April to June 1974. The material in these notes is based on Seminar Report 'Lecture notes on nuclear and L-nuclear spaces' published by Yale University, 1974. Unfortunately there are some errors and misprints there, I would like to apologize for this, and take the opportunity to put them right here.

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Y.C.W.

United College
The Chinese University of Hong Kong
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