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Proximal Flows

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PREFACE

This work is an elaboration on notes taken, mainly by Professor N. Markley, during a course entitled "Topics in topological dynamics" which was given by the author in the spring semester of the academic year 1973-74 at the University of Maryland. The main theme on which these notes are based is the notion of proximality. This notion is exploited in two principal directions. The first one is the abstract "algebraic" theory of topological dynamics, created by R. Ellis, and the other is H. Furstenberg's theory of boundaries for Lie groups and of harmonic functions. Admittedly these two theories have different flavors and use different techniques, yet we think that an interaction between them might be fruitful.

A good example of this interaction is Furstenberg's characterization of continuous harmonic functions on a symmetric space $D = \mathbb{G}/\mathbb{K}$ as those continuous functions in $L^{\infty}(D)$ whose orbit closure (in the weak $*$ topology) is a strongly proximal flow, (Theorem VI.3.1.). Other instances are the concrete identification of the universal strongly proximal flow and the generalized strong Bohr compactification of a connected semisimple Lie group with a finite center, (Theorems IV.3.2. and VIII.3.6. respectively).

The notes are divided into ten chapters each starting with a short introduction, which describes the material included and indicates its main sources. The prerequisites for reading the "pure" topological dynamics parts are just point set topology and elementary functional analysis. For the rest a certain knowledge of Lie group theory and of probability is needed. Actually only a very restricted portion of these theories is used; I tried to summarize the necessary results in sections IV.1. and V.3. If one is interested only in the abstract theory of topological dynamics and the theory of PI-flows then he can skip chapter IV, V and VI.

I wish to thank Professors R. Ellis and L. Shapiro with whom I had the pleasure of colaborating on the work "PI-flows" [12]. The major part of Chapters IX and X is based upon this paper. I wish to thank Professor W.A. Veech for several fruitful discussions and for his permission to use his unpublished result (Theorem III.4.1.). My thanks are due to all the participants of the course, and to Professors J. Auslander and R. Lipsman who read parts of the manuscript. Professor H. Furstenberg gave me many helpful suggestions during the writing of the notes which improved the whole work immensely. Finally I would like to warmly thank Professor N. Markley who originated the idea of preparing these lectures notes took notes during the course, corrected many mistakes, simplified proofs and took upon himself the painful task of supervising the printing of the manuscript.

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