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Čech and Steenrod Homotopy
Theories with Applications to
Geometric Topology



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to Marilyn and Gretchen

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