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Dedicated to
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Organizers

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INTRODUCTION

This volume may be considered as a first attempt at synthesizing the multiple relationships between the λ-Calculus and Computer Science. The volume arose from a Symposium on λ-Calculus and Computer Science Theory that was held in Rome, Italy, March 25-27, 1975, at the Consiglio Nazionale delle Ricerche, organized by the Istituto per le Applicazioni del Calcolo "Mauro Picone".

The first idea of this rather special symposium was born during a session of the European Association for Theoretical Computer Science. The main topics of the Symposium were: 1. λ-calculus models and semantics of programming languages; 2. the Church-Rosser theorem and its applications; 3. algorithms of the metatheory; 4. applicative terms as models of computation; 5. applications of typed λ-calculus.

Other topics treated were algebraic models of computations and relational caculi.

The Symposium was sponsored by the European Association for Theoretical Computer Science, the Consiglio Nazionale delle Ricerche and the Associazione Italiana per il Calcolo Automatico. The organizing committee for the Symposium consisted of C. Böhm, University of Rome and I. Galligani, Director of the Istituto per le Applicazioni del Calcolo "Mauro Picone" (IAC), Rome. The scientific committee consisted of: G. Ausiello, CSSCCA, Rome; H. Barendregt, University of Utrecht (Holland); C. Böhm, University of Rome; M. Dezani Ciancaglini, University of Torino (Italy); E. Engeler, Eidg. Techn. Hochschule, Zürich (Switzerland); G. Huet, IRIA (France); M. Nivat, University of Paris (France); M. Hyland, University of Oxford (Great Britain); D. Scott, University of Oxford (Great Britain); M. Venturini Zilli, IAC, Rome.

Coordination was cared for by M. Dezani Ciancaglini, University of Torino and M. Venturini Zilli, IAC, Rome.

Seventeen papers were presented and discussed at the Conference; three papers were not presented orally but are included in this volume. It was impossible to include the paper by K. Indermark in this volume. The few words that follow are intended to orient the reader who may be familiar with general aspects of the λ-Calculus and Computer Science Theory but not particularly with the specific work of the participants at the Symposium.

In his introductory talk D. Scott defines the nature of the relation between class abstraction and λ-abstraction in models for the λ-Calculus and gives a short critical analysis of the present state of foundational research concerning Combinatory Logic and its relationship with Predicate Logic.

The problems of least fixed points and of the semantics of programming languages are particularly stressed in De Bakker's paper in which it is stated, contrary to the application by Manna of Scott's theoretical results, that both call-by-value and call-by-name mechanisms are strictly related to the least fixed points.

In the context of Scott's work R. Nakajima introduces a generalization of normal forms considered in the framework of infinite λ-expressions.

The aim of M. Hyland's paper is to stress that, generally speaking, a partial order relation on terms of the λ-calculus possesses a characterization in terms of its computational significance and of contexts. A particular theorem is proved by H. Egli relating the meaning of typed and type-free terms in corresponding λ-calculus models over complete partially ordered sets.

The relationship between typed models and type-free models is investigated also in V. Sazonov's paper in which the author shows a particular definition of sequential and parallel functionals for these kinds of models. L. Aiello and M. Aiello seek to describe a reasonable semantics for a programming language in the context of a typed λ-calculus environment. A particular interpretation of λ-calculus in terms of a certain collection of algorithms is presented by L. Nolin through the construction of certain models very near to Scott's classical models and to the URS (Uniformly Reflexive Structures).

In the framework of algebraic languages A. Dubinsky's paper aims at defining the strict connection between the generalized automata theory and the algebraic theory of computation.

G. Ausiello's work is also related to the automata field and rewriting systems. He examines how λ-calculus can be used to describe the behavior of time-varying systems" in particular the problem of synchronization and the link with the developmental languages are analyzed.

On the other hand R. de Vrijer affirms that the abstract term system λλ presented in his paper is strictly connected to the Automath family of languages and may be considered as a simple generalization of AUT-QE.

In relation to the problems of the mechanization of first and second order theories A. Huet presents a unification algorithm for typed λ-calculus; in particular an algorithm which searches for the existence of unifiers in ω-order logic, and gives a proof of its correctness.

Another central point of interest is represented by the analysis of the URS considered as an interesting and clear axiomatization of recursion theory. H. Barendregt shows how to introduce the use of length of computation in the URS by means of the construction of a Normed Uniformly Reflexive Structures that permit us to overcome some of the defects of standard URS and rediscover some of Moschovakis' results concerning the length of computation in recursion theory.

Moreover, some important properties concerning nonterminating computations and terminating ones are presented in M. Venturini's work in which the problem of the shift from the former to the latter is analyzed.

The idea of analyzing the properties of the complexity of computations in Combinatory Logic is at the base of the work of C. Batini and A. Pettorossi who introduce various notions of computational resource and who seek to characterize levels of subrecursiveness in Combinatory Logic. A particular definition of the concept of "subbase" is given and some results about the generative and computational power of subbases are shown.

In this manner these last papers stress the particular relationship between Combinatory Logic and Recursion Theory.

Lastly some aspects of the pure λ-calculus are revisited in order to extract some computational properties or some technical results.

So J.J. Lévy aims to prove by means of introduction of a labelled λ-calculus Welch's conjecture about the completeness, in the reducibility sense, of inside-out reductions. Then the general problem of inside-out reductions and continuous semantics is examined in P.H. Welch's paper in the light of the consideration that the natural meaning of an expression is just the union of "instant meanings".

The work of C. Böhm and M. Dezani Ciancaglini presents a particular partition of the set of λ-terms in $2\omega + 1$ classes considered as a natural kind of type introduction in which every λ-term possesses a unique type. This classification can be effectively determined only for λ-terms in normal form.

From another point of view the purpose of G. Jacopini is to define a necessary and sufficient condition by which two combinators can be identified without introducing contradictions with the axioms of

Combinatory Logic.

J.W. Klop gives an easier version of the main lemma of a well-known theorem by H. Barendregt concerning the solvability of λ-I-terms.

In his final talk D. Scott outlines an informal survey of the wide variety of points of view issuing from the papers presented at the Symposium, in an attempt to establish, on a philosophical basis, some common measure of critical comparison without any pretence to furnish a definitive philosophy of combinators.

At the end of the volume Barendregt presents, as a stimulating challenge, a set of open problems with his critical comments.

Particular thanks are due to A. Faedo, President of the Consiglio Nazionale delle Ricerche, for providing assistance which contributed in large measure to the realization of the meeting. Special thanks go to I. Galligani, Director of the IAC, which provided the financial support, the clerical staff and all other organizing facilities.

I would also like to thank the members of the Scientific Committee for their help in the management of the congress. In particular I would like to express my gratitude to M. Venturini Zilli for her invaluable help and intelligent suggestions.

Corrado Böhm

Roma, June 1975