

# Lecture Notes in Computer Science

Edited by G. Goos and J. Hartmanis

15

---

L Systems

Edited by Grzegorz Rozenberg and Arto Salomaa

---



Springer-Verlag  
Berlin · Heidelberg · New York 1974

**Editorial Board:** P. Brinch Hansen · D. Gries  
C. Moler · G. Seegmüller · N. Wirth

Prof. Dr. Rozenberg  
Mathematisch Instituut  
Rijksuniversiteit te Utrecht  
Utrecht/Holland

Prof. Dr. A. Salomaa  
Institute of Mathematics  
University of Aarhus  
Dept. of Computer Science  
Ny Munkegade  
8000 Aarhus/Denmark

**Library of Congress Cataloging in Publication Data**

Main entry under title:

L systems.

(Lecture notes in computer science, 15)

Most of the papers were presented at a conference  
in Aarhus, Denmark, Jan. 14-25, 1974.

Bibliography: p.

I. L systems. I. Rozenberg, Grzegorz, 1934- ed.  
II. Salomaa, Arto, ed. III. Series.  
QH491.L2 574'.028'5424 74-16417

---

AMS Subject Classifications (1970): 68A30, 92A05, 68A25, 94A30,  
02F10

CR Subject Classifications (1974): 5.22, 5.23

---

ISBN 3-540-06867-8 Springer-Verlag Berlin · Heidelberg · New York  
ISBN 0-387-06867-8 Springer-Verlag New York · Heidelberg · Berlin

This work is subject to copyright. All rights are reserved, whether the whole  
or part of the material is concerned, specifically those of translation,  
reprinting, re-use of illustrations, broadcasting, reproduction by photo-  
copying machine or similar means, and storage in data banks.

Under § 54 of the German Copyright Law where copies are made for other  
than private use, a fee is payable to the publisher, the amount of the fee to  
be determined by agreement with the publisher.

© by Springer-Verlag Berlin · Heidelberg 1974. Library of Congress  
Catalog Card Number 74-1641-7. Printed in Germany.

Offsetdruck: Julius Beltz, Hemsbach/Bergstr.

## PREFACE

Since the introduction of developmental languages in 1968 by A.Lindenmayer ("Mathematical models for cellular interactions in development", Parts I and II, Journal of Theoret.Biology, 1968, v.18, 280-315), this field, now commonly referred to as the study of L systems, has been very active. The number of people interested in this area has become larger and larger, and the yearly growth in the number of papers has so far been exponential. One reason for this widespread interest seems to be that this field is able to attract people with quite different backgrounds. For instance, automata and formal language theorists, like the present editors, have found in L systems an interesting and fruitful alternative to the ordinary Chomsky type of grammars.

From the biological point of view, L systems have provided a useful theoretical framework within which the nature of cellular behaviour in development can be discussed, computed and compared. Their study has also provided a number of biologically interesting results.

From the mathematical point of view, L systems have opened a new dimension in formal language theory. Its novelty is reflected in both new types of problems and new techniques for solving them.

For a survey of biological aspects of the theory the reader is referred to the contribution by A. Lindenmayer to the book "Developmental systems and languages" by G.T. Herman and G. Rozenberg (North Holland Publ. Company, 1974). The first paper in the present volume serves as a survey of the mathematical theory of L systems. It is hoped that the remaining ones give an idea of the current research. In this sense the above mentioned book and this volume should complement each other: the first being more tutorial and the second more research oriented.

The division of papers into sections is not intended to be too conclusive. It just provides a division line which might help the reader. As far as referencing is concerned, a common bibliography is provided at the end of the volume. The reference numbers in the individual papers refer to this bibliography.

Most of the papers in this volume were presented at a conference in Aarhus, January 14-25, 1974. The conference was the third in a series of Open House meetings arranged by the Computer

#### IV

Science Department of the University of Aarhus. The topics of the first two meetings in 1972 and 1973 were: "unusual automata theory" (proceedings available from the department) and "semantics of programming languages" (no proceedings). This time the topic was L systems, and most of the people active in this area were present.

The editors express their gratitude to all the participants, as well as to Statens Naturvidenskabelige Forskningsrad in Denmark for financial support

UTRECHT, May 1974

Grzegorz Rozenberg      Arto Salomaa

## CONTENTS

A SURVEY OF MATHEMATICAL THEORY OF L SYSTEMS	1
Theory of L systems: from the point of view of formal language theory	
G. Rozenberg	1
BIOLOGICAL ASPECTS OF THE THEORY OF L SYSTEMS	24
A model for the growth and flowering of <u>Aster novae-angliae</u> on the basis of table $\langle 1,0 \rangle$ -systems	
D. Frijters and A Lindenmayer	24
Adding continuous components to L-systems	
A. Lindenmayer	53
Formal language theoretical approach to intracellular behaviour	
V. Bédian and G.T. Herman	69
TOWARDS CHARACTERIZATION RESULTS	72
Three useful results concerning L languages without interactions	
A. Ehrenfeucht and G. Rozenberg	72
On the size of D0L languages	
P.M.B. Vitányi	78
Generatively deterministic L languages. Subword point of view	
A. Ehrenfeucht, K.P. Lee and G. Rozenberg	93
GROWTH FUNCTIONS	104
Growth of strings in context dependent Lindenmayer systems	
P.M.B. Vitányi	104
Some growth functions of context-dependent L-systems	
J. Karhumäki	127
D0L systems with rank	
A. Ehrenfeucht and G. Rozenberg	136
NEW ATTACKS ON D0L EQUIVALENCE PROBLEM	142
Equivalence of L-systems	
M. Nielsen	142
The syntactic inference problem for D0L-sequences	
P.G. Doucet	146

## VI

Free groups in Lindenmayer systems P. Johansen and E. Meiling	162
DIFFERENT WAYS OF DEFINING L LANGUAGES	177
Notes on pre-set pushdown automata	
J. van Leeuwen	177
Recurrence systems	
G.T. Herman	189
Adult languages of L systems and the Chomsky hierarchy	
A. Walker	201
Structured OL-systems	
K. Čulik II	216
Context in parallel rewriting	
K. Čulik II and J. Opatrný	230
Nonterminals and codings in defining variations of OL-systems	
S. Skyum	244
LOOKING FOR A GENERAL FRAMEWORK	250
Iteration grammars and Lindenmayer AFL's	
A. Salomaa	250
Hyper-AFL's and ETOL systems	
P.A. Christensen	254
$\Omega$ -OL systems	
A.L. Szilard	258
Bounded parallelism and regular languages	
D. Wood	292
Multidimensional Lindenmayer organisms	
B.H. Mayoh	302
BIBLIOGRAPHY	327
Bibliography on L systems	
K.P. Lee and G. Rozenberg	327