

# Lecture Notes in Mathematics

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Hui-Hsiung Kuo

Gaussian Measures  
in Banach Spaces

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## PREFACE

This monograph is based on the lecture notes of a course entitled "Applications of Measure Theory" given in the Spring of 1974 at the University of Virginia. As the reader can easily see, the material delivered does not cooperate with the course's title very well. Our primary object in this course was to give an introduction to the notion of abstract Wiener space and study some of the related topics. We covered the first two chapters and the first three sections of Chapter III. The last four sections were added in when I rewrote the lecture notes. I deeply regret that in this course we did not discuss in details the recent works of J. Eells, K.D. Elworthy and R. Ramey, among others on the integration on Banach manifolds. I feel that it would be too ambitious to include their works in these notes.

I would like to express my appreciation to Professor Leonard Gross and Professor Kiyosi Ito for their constant encouragement and mathematical influence. The conversations with them have always been a source of inspiration. I would like to thank Tavan Trent for proof-reading parts of the manuscript. My special thanks go to Barbara Smith and Fukuko Kuo for typing the manuscript. The preparation of these notes is partially supported by the National Science Foundation.

H. -H. Kuo

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