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Nathan Jacobson

PI-Algebras

An Introduction



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Prof. Nathan Jacobson
Dept. of Mathematics
Yale University
Box 2155, Yale Station
New Haven, CT 06520/USA

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Foreword

These are lecture notes for a course on ring theory given by the author at Yale, September - December, 1973. The lectures had two main goals: first, to present an improved version of the theory of algebras with polynomial identity (over a commutative coefficient ring) based on recent results by Formanek and Rowen and second, to present a detailed and complete account of Amitsur's construction of non-crossed product division algebras.

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