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Abelian Harmonic Analysis,
Theta Functions and Function
Algebras on a Nilmanifold



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PREFACE

These notes are concerned with the inter-relationship between abelian harmonic analysis, theta functions and functional analysis on a certain nilmanifold. Some of the results in these notes are new and some are old. However, our approach, because it puts a certain nilmanifold and its function theory at center stage, often leads to new proofs of standard results. For example, we view theta functions as the analogue on a nilmanifold of the spherical functions on the sphere, where the Heisenberg group plays the role of the orthogonal group. Thus the classical theta identities will follow from basic group theoretic results.

Historically, there are many names that can be associated with the topics treated in these notes. Because of the informal nature of these notes we have not made any effort at giving complete biographical references for results and have given only references to the sources we ourselves have used. If we have overlooked someone's work or state a result without reference that someone knows to be his, we apologize in advance. However, we would be less than honest if we did not admit the great influence of the ideas of J.Brezin, G.W.Mackey and A.Weil on our work. Indeed, after so many years of talking with Brezin many of the ideas or germs of ideas in these notes may be his. In addition, we should also mention the work of Cartier which stands somewhere in the middle ground between the work of Weil and that presented in these notes.

It may be advisable at this point to explain to the reader how the material in these notes has been labelled. The reader will find some material labelled Chapter and some material labelled Appendix. The material with sections labelled Chapter is more complete and well rounded. The material labelled Appendix is of a more tentative nature and does not yet seem to have taken on a definitive form. The last appendix has its own list of references and the other appendices and chapters have a single list of references located on page 74.

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