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Representations of Commutative Semitopological Semigroups

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Preface

Harmonic analysis is primarily the study of functions and measures on topological spaces which also have an algebraic structure. In this book, the underlying structure is given by a commutative associative separately continuous multiplication, that is, a commutative semitopological semigroup (CSS). Everybody knows that representation theory is useful in studying almost all (maybe even all) mathematical structures. The theory of representing CSS's in compact CSS's is essentially the same as the theory of weakly almost periodic functions (see Eberlein [1], de Leeuw and Glicksberg [1], Berglund and Hofmann [1]).

To discover more structure, we investigate representations of CSS's in objects native to harmonic analysis. In order of increasing generality, they are the unit disc, the unit ball in an L^∞-space, the unit ball of the quotient of a function algebra, and the unit ball in the algebra of bounded operators on a Hilbert space. The latter three are furnished with weak topologies in which multiplication is separately continuous and the unit balls are compact.

This point of view provides a unified framework for diverse ideas like semicharacters, positive-definite functions, completely monotone functions, the Hausdorff moment problem, functions of bounded variation, the Plancherel theorem, duality for locally compact abelian groups, representation of uniquely divisible CSS's (the theory of Brown and Friedberg [1]), dilation theory on Hilbert space, et cetera. Interesting new problems can be posed even for such seemingly trivial semigroups as $X \cup \{0\}$, X any set, xy = 0, x0 = 0 $(x,y \in X)$.

We hope that this book will be interesting to both commutative harmonic analysis and topological semigroup people. As Berglund and Hofmann [1] point out, functional analysis is important in studying semitopological, rather than topological, semigroups, so we expect the reader to be acquainted with basic functional analysis (Rudin's book [2], for example). Some knowledge of duality theory for locally compact abelian groups would be helpful.

A substantial part of the work is new. Unless otherwise stated, semigroups will be assumed to be commutative, semitopological, and written multiplicatively. The usual exceptions to the latter are Z, Z_+, $\mathbb{R}$, $\mathbb{R}_+$ (integers, nonnegative integers, reals, nonnegative reals respectively) and their Cartesian products, which will always have the additive structure. Indices for authors, subjects, and symbols are provided.

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C.F.D.

D.E.R.

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