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Edited by J. Ehlers, München, K. Hepp, Zürich,
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A. H. Völkel

Fields, Particles and Currents



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Author**A. H. Völkel**

Pädagogische Hochschule Berlin

Malteserstraße 73-100

1000 Berlin 46/BRD

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Symbols

$a:=b$	a is defined by b
\cup, \cap	Union, Intersection
\wedge	Logical "and"
M^T	Transposed matrix
$(\dots)^T$	Truncated vacuum expectation value or Wightman function
φ^*	Adjoint Operator
\bar{f}	Complex conjugate c-number or adjoint spinor ($\bar{\psi}^T \gamma^0$)
B^\dagger	Adjoint spinor operator (B^{*T}) or complex conjugate and transposed matrix (\bar{B}^T)
D^\perp	Set of vectors orthogonal to the set D
\hat{S}_n	Symmetrization with respect to n
\hat{A}_n	Antisymmetrization with respect to n
$\varphi^\#$	Occurrence of either φ or φ^*
$\hat{\varphi}$	Closure of the operator φ

Introduction

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The principle intention of this manuscript is to provide an introduction to the basic ideas and results of relativistic quantum field theory for students intending to work in this topic of mathematical physics, physicists with some intellectual interest in this field and mathematicians who just want to glance beyond the fence. We have aimed at a level of presentation which is intelligible to graduate students who are familiar with quantum mechanics, the classical theory of the electro-magnetic field and with the elements of Hilbert space theory.

The manuscript is based on lectures and seminars given by the author at the physics departments of the Katholieke Universiteit Nijmegen (The Netherlands), F. U. Berlin, Universität Bielefeld (West-Germany) and Pontificia Universidade Catolica do Rio de Janeiro (Brasil). The material presented is a selection due to the author's taste. It goes beyond the classical text books "PCT, Spin and Statistics, and all that" by R. F. Streater and A. S. Wightman and "The General Theory of Quantized Fields" by R. Jost by including a variety of results -- Carruthers' Theorem, various weak asymptotic conditions respectively series of fields as well as current generated global charges and symmetries respectively their breaking -- which have been gained during the decade after the publication of these books. On the other hand most important topics like constructive quantum field theory [Can 1 , Gli 1-6 , Si 2 , Ve 1] could not even be touched for the lack of space and also for the limited strength of the author. The same has to be remarked with respect to general properties of the S-matrix [Mart 1 , So 1]. Hence the selection does not represent any judgment. The same applies to the quoted literature, which depends strongly on the author's memory.

In the first part we present the general back ground ideas of the notion of a relativistic quantized field. Moreover the main mathematical tools needed later on are shortly reviewed. In order to gain some practice and familiarity with them an intensive treatment of free quantized fields is added.

Part II starts with the setting of the general frame of what is now a days called a local quantum field theory or Wightman field theory. The assumptions of Wightman are transcribed into properties of vacuum

expectation values of the fields and their analytic continuations. They in turn serve to derive the main results about the structure of the theory -- PCT- and Carruthers' Theorem, Spin-Statistics, Cluster Theorems etc.

In part III the asymptotic particle interpretation and the scattering matrix are constructed. Moreover the various weak asymptotic conditions and the asymptotic series of fields are derived.

Part IV contains the construction of global charge operators and symmetries from locally conserved currents as well as the description of their spontaneous breaking.

In an appendix the generalization to non tempered (localizable) Jaffe fields is indicated.

This work would not have been possible without the help and encouragement of many people. Many critical questions of the participants of the lectures as well as numerous clarifying discussions with colleagues, especially with M. Nagels, T. Rijken, B. Schroer, J. J. de Swart, J. A. Swieca and Uta Völkel paved the way for this manuscript. To all of them I am very grateful. I am indebted to Bert Schroer who guided my first steps to quantum field theory and whose permanent enthusiasm kept alive my interest in it through all the years.