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Maximum Probability Estimators and Related Topics

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PREFACE

In the last few years, the authors have, in a number of papers, been developing the theory of maximum probability estimators. This theory is a comprehensive one of asymptotically efficient estimators, and, as the reader will easily verify below, it is not, as has been sometimes incorrectly stated, limited either to the non-regular case or to errors of estimation of order $n^{-1/2}$. On the contrary, it includes both maximum likelihood theory and the non-regular case as special cases. The present monograph is intended as an introduction to the theory which could be studied by graduate students working by themselves or in a seminar. It is largely self-contained, gives some important proofs in greater detail, brings together material from a number of papers, and provides supplementary discussion not present in the papers. The appendix contains a number of non-trivial illustrative examples; more will be found in the original papers. The monograph is devoted almost entirely to the case where θ, the parameter being estimated, is a scalar, but the appendix contains a number of examples where θ has several components. These will cause the reader no difficulty, and it is hoped that they will encourage him to go to the original papers for, inter alia, the multidimensional theory. The monograph does not exhaust the contents of the papers.

Chapter 7 discusses the application of the basic theory to the problem of testing hypotheses. It extends earlier joint work of both authors, but was written by L. Weiss. The rest of the monograph can be read with no reference to Chapter 7.

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TABLE OF CONTENTS

CHAPTER 1 :	Purpose of this Monograph	1
CHAPTER 2 :	The Maximum Likelihood Estimator	5
CHAPTER 3 :	The Maximum Probability Estimator	12
CHAPTER 4 :	Maximum Probability Estimators with a General Loss Function	30
CHAPTER 5 :	Asymptotic Behavior of the Likelihood Function. Asymptotically Sufficient Statistics.	40
CHAPTER 6 :	Efficiency of Maximum Likelihood Estimators	51
CHAPTER 7 :	Testing Hypotheses	61
	Appendix	93
	References	105