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PREFACE

The Sydney Category Theory Seminar was born in the middle of 1972, when the category-theorists at the three universities in Sydney conceived the idea of meeting for a whole day each week. The papers collected in this volume represent a large part of the mathematics that has emerged from its first eighteen months. The title-date "1972/1973" represents accurately enough the time at which the work was done, although the final details of it, along with the writing-up and the typing, have brought us past the middle of 1974.

As editor I was excessively sanguine about the time it would take to turn ideas into theorems; for the unforeseen delays I owe apologies to Springer-Verlag and especially to Brian Day, whose first two papers were received more than a year ago.

The papers below are connected by more than our local proximity; the common theme is that of categories with structure. If categories are seen as analogous to sets, then pure category theory is the analogue of pure set theory, although a lot richer because of the "increase in dimension by 1". But then the analogue of sets with structure is categories with structure; and monoidal categories, monoidal closed categories, enriched categories, categories bearing a monad, and so on, become to the category theorist what his groups, rings, fields, and modules are to the algebraist.

If monoidal closed categories are in some sense the "fields" of category theory, then Brian Day is our field theorist. In fact his papers are concerned with two types of structure - monoidal and especially monoidal closed ones, and adjunctions - and with the relations between these two structures; all this moreover at the "enriched" level. His first paper gives a new adjoint-functor theorem, which has since been adapted by Mikkelsen to the elementary topos situation, and which may be seen as replacing by a two-step process the transfinite

tower construction of Applegate and Tierney; along with applications to completions and to monoidal completions. His second combines his earlier work on monoidal closed structures for functor categories, and on the reflexion of monoidal closed structures, to study monoidal closed structures on reflective subcategories of functor categories; and gives a great many concrete applications. His third discusses the embedding of a non-monoidal closed category into a monoidal one, of great value for coherence problems, with extensions to bicategories; and his fourth uses his techniques to look deeply at certain cartesian closed categories of topological spaces. The reader will quickly perceive the definitive elegance of his theorems.

My own papers correspond in some sense to universal algebra. As equational algebras of any given species correspond to a monad on the category of sets - or on a more general category if we want things like topological groups - so categories with a given species of equational structure correspond to a 2-monad, also called a doctrine, on the 2-category of categories, or more generally on any 2-category. In this setting the "coherence problem" is that of finding the doctrine explicitly from a knowledge of its algebras. My first paper looks at a subclass of doctrines that admit a very concrete representation by what I call clubs, and a smaller subclass where the coherence problem can be formulated as a word-problem. My second paper concerns the interplay between equational structures and adjunctions; and my third gives some coherence results at the doctrine level, getting the fuller results available in the club case by specialization. The reader would do well to read the first paper last, except for §1 and §10, which alone are used in the last two papers.

The elementary 2-categorical background needed both for my papers and for Street's is contained in a joint expository paper. Street goes on in his papers to look more deeply at 2-categories. In one sense his papers are the most general; in another, the most

fundamental. If the study of various structures borne by a set has led to category theory, that of structures borne by a category leads inexorably to 2-category theory. Much of category theory can be done inside any 2-category, and the arguments are then often more transparent. Street's first paper looks at some things that can be done in a representable 2-category - essentially the same thing as a finitely 2-complete one, except that it need not contain a terminal object, which is seldom needed. In particular he studies fibrations and bifibrations at this level, along with such things as pointwise Kan extensions. His second paper, which uses the first, investigates 2-categories with a structure so rich that we can imitate those arguments, including the Yoneda lemma, that depend upon the hom functor. Even applied to the 2-category of categories, it provides new proofs and thus contributes to the "elementary" theory of categories; applied to the 2-category of ordered objects, it throws new light on elementary topoi.

The investigations of Street and Kelly, at least, are to some degree tentative, and they mention many outstanding problems which may be of interest to others.

Because of the time the volume has been in preparation, I believe it appropriate to give dates of reception for the papers (although I don't quite know what it means to "receive" my own). The order is that of the table of contents. Day: Feb. 1973 revised May 1973; Apr. 1973; Feb. 1974; Feb. 1974. Kelly-Street: Oct. 1973. Street: July 1973; Feb. 1974. Kelly: Nov. 1973; Jan. 1974; May 1974.

G.M. Kelly
19 July 1974

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