

Lecture Notes in Mathematics

An informal series of special lectures, seminars and reports on mathematical topics

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Linear elliptic
differential systems and
eigenvalue problems

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These Notes contain the lectures I had the pleasure of delivering as Visiting Professor at the Department of Mechanics of The Johns Hopkins University on the invitation of Professor Clifford Truesdell.

They are intended to be an introduction to the modern approach to higher order elliptic boundary value problems and related eigenvalue problems.

I am deeply grateful to Dr. Warren Edelstein for his kind collaboration in checking both the English and the Mathematics of these Notes.

G. Fichera

Baltimore, Md. - May 1965.

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