

Lecture Notes in Mathematics

A collection of informal reports and seminars

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Topological Vector Spaces
and Algebras



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INTRODUCTION

The lectures associated with these notes were given at the Instituto de Matemática Pura e Aplicada (IMPA) in Rio de Janeiro, during the local winter 1970. To emphasize the properties of topological algebras, the author had started out his lectures with results about topological algebras, and introduced the linear results as he went along.

The present exposition is more systematic. It is more or less divided into two parts. The first part contains the first four chapters and could be headed Linear Theory. The second part contains the five remaining chapters and would be headed Algebras.

The author has not intended to write a complete survey of the subject.

Locally convex spaces have become classical. Nuclear spaces are not used in Part II. Both theories have been omitted. Part I mainly contains structures which are not classical, but which are useful in the development of topological algebra theory according to the author's experience. Other mathematicians will feel that this author has been unfair to their own pet theories ...

A limited number of interesting constructions applicable to topological algebras are developed in Part II. It is hoped that the reader will get an idea of the tools with which one can tackle problems involving topological algebras, also of the problems which can be handled in this way. But the reader must be aware that many interesting results will not be found in these notes.

Most chapters are concluded by a section "Notes and Remarks", in which the author speaks about the history, the development of the subject. These Notes are not 100 % reliable. But the author belongs to the old-fashioned school, he believes that the history of a subject is part of the context in which the subject should be placed. An unreliable historical survey is better than none. The amount of

work that goes into the writing of a reliable historical survey is incompatible with the "Lecture Note" idea. This does not mean that the author should not try to help the mathematician who later wishes to write a reliable historical survey.

All Lecture Note authors have to meet deadlines, and decide to leave out material which definitely should be included. This is a heart-rending experience.

N.Adasch [1], [2], [3] has proved nice results about non locally convex, ultrabarreled spaces, and about the closed graph theorem "à la Ptak". These results would fit very well in the first chapter.

The author would also have included some personal results about the density, the admissibility problem of V.Klee [44], [45]. The problem has also been studied by A.H.Shuchat [70]. The author owes references to R.Retherford, these will be very useful to him when he has the leisure to write out his results.

The author must acknowledge support from the Organization of American States during his stay in Rio de Janeiro. His stay at IMPA was quite agreeable. The friendly atmosphere, the working conditions were conducive to scientific work. The scheduling of the lectures was leisurely enough, but not too leisurely.

These lecture notes have been typed by the secretaries of the Departments of Mathematics of the Université Libre de Bruxelles and the Vrije Universiteit Brussel. The author is especially grateful to the U.L.B. and its mathematics department secretaries, who did a large part of the job, since he does not belong to that institution.

Dr Ivan Cnop corrected a first draft of these notes. His help was greatly appreciated.

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