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# Lecture Notes in Mathematics

A collection of informal reports and seminars

Edited by A. Dold, Heidelberg and B. Eckmann, Zürich

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## Analytic Theory of Differential Equations

The Proceedings of the Conference  
at Western Michigan University, Kalamazoo,  
from 30 April to 2 May 1970

Edited by P. F. Hsieh and A. W. J. Stoddart,  
Department of Mathematics, Western Michigan University

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## FOREWORD

This book contains the proceedings of the Conference on Analytic Theory of Differential Equations, held at Western Michigan University, Kalamazoo, from 30 April to 2 May 1970. The manuscripts of the lectures are published as supplied by the speakers, apart from changes in format and obvious corrections.

The analytic theory is one of the oldest areas in differential equations still active. Our objective in the Conference was to bring together as many workers in the area as possible. The topics were not restricted to purely analytic theory; Part I contains closely related topics in ordinary differential equations and Part III other topics in partial differential equations.

We wish to thank the following for their contributions to the success of the Conference:

The speakers and other participants.

The National Science Foundation, for financial support.

Western Michigan University, for financial support and use of facilities.

Professor A. Bruce Clarke, Head of the Department of Mathematics, for advice and support.

The session chairmen, including Professors Y. Alavi and D.P. Giesy.

Professor Y. Alavi, for assistance in organization and proofreading.

Undergraduate assistants Scott Marovich and Tim Husband.

The secretaries of the Department of Mathematics .

In addition, we thank the following for their part in the publication of this book:

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P.F. Hsieh

A.W.J. Stoddart

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