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Linear Determinants with Applications to the Picard Scheme of a Family of Algebraic Curves

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INTRODUCTION

This paper grew out of a study of A. Weil's construction of the Jacobian variety of an algebraic curve.

The first delicate problem in Weil's construction is a rationality question connected with the symmetric product of a curve, which led me to the theory of linear determinants as exposed in Chapter I. This theory studies a 1-dimensional integral representation, called ld, of $M_{n,Z}^{(n)}$ which denotes the n-fold symmetric product of the $n \times n$ matrices i.e. the algebra of tensors in $M_{n,Z} \otimes \ldots \otimes M_{n,Z}$ (n factors) invariant under the standard action of the symmetric group on n letters. This representation aside from its geometric aspects provides a linearization of Galois theory different from the one usually advocated. A relation of ld to Azumaya algebras is just touched upon. I have kept this chapter entirely in the notion of commutative algebra since I hope it has an independent interest.

In chapter II is studied the geometric aspect of ld which is first of all the following: Let $f : X \to Y$ be a finite morphism of schemes whose fibers have constant rank n and let $X_Y^{(n)}$ denote the n-fold symmetric product of X over Y. By means of ld is constructed a section of $X_Y^{(n)}$ over Y which underlies the geometric map which to a geometric point y of Y associates the unordered

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n-tuple $\{x_1,\ldots,x_n\}$ where $x_1,\ldots,x_n$ denotes the geometric points of X lying over y each repeated as many times as the ramification index prescribes. This is applied to prove that the n-fold symmetric product of a flat family of non singular curves represents the functor "n-fold sections". This theorem solves the rationality question mentioned above. In case of a projective family of curves "n-fold sections" becomes relative Cartier-divisors and the theorem ties up with the theory of Chow points and Grothendiecks theory of Hilbert Schemes.

The goal of Chapter III and IV is to generalize Weil's construction of the Jacobian variety of a single curve to construct Picard schemes, in the sense of Grothendieck, for a flat and proper family of geometrically reduced and irreducible curves.*) The result we get is that the Picard scheme of such a family exists after a faithfully flat, finite type extension of the parameter scheme. Recent results of M Raynaud [Ra]**) p. 178 allow to descent the obtained Picard schemes in the case where either the parameter scheme is 1-dimensional or the case where all fibers are non singular. Hopefully, future results along the lines

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*)The additional technical assumptions are: the base scheme is noetherian and the family satisfies condition II.1.1.

**)Square brackets contain references to the Bibliography at the end of the paper.

of Raynaud will allow to descent the result in general.

In case of a projective family of curves the result is a special case of Grothendieck's general existence theorem for the picard schemes of a projective family.

The main tools in the construction given here are M. Artin's generalization of Weil's theorem on the construction of a group from a rational group law and of course Grothendieck's theorems on cohomology of coherent sheaves, especially those of base change type. It should also be mentioned that the possibility of allowing singular fibers in this construction was opened up by theorems of Rosenlicht [R_2]. Finally a new (I believe) version of the seesaw principle is instrumental, a proof of this version in its full generality is included in Chapter IV.

This material has been presented at a seminar at M.I.T.; as a result I have broadened the paper with a few proofs of older results due to Weil and others. I would like to take the opportunity to thank Connie Clayton for her careful and fast typing of the manuscript.

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B.I.