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Multidimensional Inverse Problems for Differential Equations

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INTRODUCTION

An inverse problem for a differential equation is any problem involving the determination of the coefficients or right-hand side of a differential equation from certain functionals of its solution.

Two significant advances have been made previously in the study of inverse problems for differential equations.

The first is in inverse problems for STURM-LIOUVILLE equations ([1], [13], [19], [23], [36]). In these problems, the coefficient in a second-order differential operator is required to be found from the spectral function of the operator. In [19] and [2], a number of problems involving the determination of the coefficients of a partial differential equation are shown to be reducible to inverse problems for STURM-LIOUVILLE equations. It is assumed there that the coefficients are functions of a single variable.

The second is in problems of potential theory ([17], [20], [25], [29], [33]). In the inverse problems of this type, the right-hand side of an elliptic partial differential equation has to be determined. Highly restrictive additional conditions are imposed on the right-hand side. Thus, in [17] and [20], the required right-hand side in POISSON'S equation is assumed to be a function having values 0 and 1 only, and the set where it is 1 is a star-shaped domain. The same sort of restrictions are also imposed on the right-hand side in other papers dealing with the inverse problem of potential theory.

Until now, multidimensional inverse problems have been given comparatively little consideration. In a multidimensional problem, the required coefficients or right-hand sides of the differential equations are generally arbitrary functions of several variables belonging to a certain function space.

Multidimensional problems were first investigated in the papers of Ju. M. BERZANSKII. In [4] a uniqueness theorem was proved for the solution to the inverse problem for SCHRÖDINGER'S equation in the class of piecewise analytic functions. In [8] and [9] functions were constructed for some multidimensional inverse problems of quantum scattering theory that are similar to the GEL'FAND-LEVITAN functions occurring in the inverse problem for the STURM-LIOUVILLE equation.

This monograph investigates a number of multidimensional inverse problems whose formulations differ from those of the papers mentioned above. A portion of the results have been published as short notes ([21], [22], [30], [31]).

A characteristic aspect of multidimensional inverse problems is their property of not being well-posed in the sense of HADAMARD. Thus, it is advantageous to make use of the general notions and approaches to ill-posed problems developed in [16], [20], [34], and [35]. Central to the theoretical study of a problem that is not well-posed in the sense of HADAMARD is the proof of a uniqueness theorem. The monograph consists primarily of proofs of uniqueness theorems for the formulations in question.

The inverse problems for which uniqueness theorems will be proved are linear and their solution is reduced to the solution of first-order linear operator equations. Thus algorithms may be constructed to solve them numerically by application of the general methods developed in [20] and [35] for linear equations.

The methods used to prove uniqueness lead to special algorithms for the problems. They also make possible the derivation of estimates characterizing the stability of the various formulations on certain specific compact sets (for example, the set of functions whose gradients are uniformly bounded).
We shall confine ourselves here to the study of inverse problems for second-order differential equations although some of the methods carry over to higher-order equations [18].

The inverse problems considered for hyperbolic equations are reducible to problems of integral geometry and so Chapter 1 is devoted to some aspects of it. Chapter 2 establishes uniqueness theorems for the solution to the inverse problem for the telegraph equation with the help of the results of Chapter 1 while Chapter 3 does the same thing for the wave equation.
Chapters 4 and 5 deal with inverse problems for the heat equation and for elliptic equations. They are reduced to the solution of certain integral equations of first kind.
In Chapters 3 and 4, some applied problems are discussed that lead to corresponding versions of the inverse problem.