Lecture Notes in Mathematics

A collection of informal reports and seminars
Edited by A. Dold, Heidelberg and B. Eckmann, Zürich

165

Joel M. Cohen

University of Pennsylvania, Philadelphia, PA/USA

Stable Homotopy

Springer-Verlag
Berlin · Heidelberg · New York 1970

 Library of Congress Catalog Card Number 77-139950. Printed in Germany.
Title No. 1952

Offsetdruck: Julius Beltz, Weinheim/Bergstr.

Introduction

These notes are essentially the lecture notes of a course I gave at the University of Chicago in the summer of 1968.* Most aspects of stable homotopy are touched on and some are studied in very great detail. It should, however, be emphasized that we are only concerned with _finite_ CW complexes. Thus one never has to worry about the problems which may arise for infinite CW complexes; i.e. certain long exact sequences which are easy to get for finite dimensional CW complexes become very difficult in general unless one takes great care in defining the morphisms (as J. M. Boardman has done in his Warwick lecture notes; or see _Tierney_).

It is assumed that the reader has had a year of algebraic topology (a course which covers the equivalent of most of _Spanier_, say). I quote without proof some theorems from first year topology (e.g. the Hurewicz theorem) and prove others. In addition I assume the reader has some understanding of spectral sequences and what they can do. Specifically, I assume existence of the Serre Spectral Sequence in homology. _Spanier_ covers quite adequately the necessary material.

For the computations of the stable homotopy groups of spheres in Chapter V, I quote a lot of results on the Steenrod Algebra--all of which can be found in _Steenrod-Epstein_ or _Mosher-Tangora_. Lack of prior knowledge of cohomology operations will not interfere with the understanding of this section, although the reader may have to accept some results on faith (or study the above-mentioned books).

This set of notes has a quite different point of view on the whole from Frank Adams' lecture notes on stable homology. I feel

*The author was partially supported by the National Science Foundation during the preparation of these notes.

that to some degree, these complement the other. Although I do construct the Adams spectral sequence for completeness, not very much is said about it here and the reader is encouraged to pursue the subject either in Adams' notes or in Mosher-Tangora. The present method of computing the stable homotopy groups of spheres is somewhat simpler than the Adams spectral sequence in the dimensions where it is done. (Higher up this method seems to break down and the Adams method is much neater.)

Chapter IV, on stable homotopy and category theory is entirely the work of Peter Freyd. The proofs are to some extent my own--I tried to make them more topological than category theoretical where possible; but the fact remains that the main results, which are purely topological statements, cannot be proved without using (or directly mimicking) Freyd's embedding of the stable homotopy category into an abelian category.

Thanks are due many people for the ideas incorporated in these notes. My interest in the subject was aroused by George Whitehead; much of my thinking was influenced by him and several proofs are lifted directly from him. Chapter V is an abridged version of my thesis written under Donald Anderson. I express my deep gratitude to him for many helpful suggestions during the original writing and since. In addition many parts of these notes grew out of very useful discussions with Frank Peterson, David Kraines, Gerald Porter, Peter May, Peter Freyd and Brayton Gray. I wish to thank Susan McMahon, Mary Vallery and Cecelia Ricciotti for putting up with my handwriting and typing this manuscript.

TABLE OF CONTENTS