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Seminar on Algebraic Groups and Related Finite Groups

Held at The Institute for Advanced Study
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INTRODUCTION

This volume contains the Notes of a seminar held at the Institute in 1968-69, in the framework of a program on linear algebraic groups and finite groups.(*) They emphasize a borderline topic between these two topics: the linear representations, both modular and over the complex numbers, of the finite Chevalley groups and of some generalizations of them. They also contain a discussion of some questions on algebraic groups which are both relevant to the main theme and of independent interest.

Part A first supplies some background material: construction and main properties of Chevalley groups over a field or over $\mathbb{Z}$, and of rational representations of a semi-simple algebraic group over an algebraically closed groundfield. It then turns to results of Curtis and Steinberg describing the representations of a Chevalley group over a field of non-zero characteristic whose differential is also irreducible, and the construction of the irreducible representations of the group by means of them. As far as the finite Chevalley groups are concerned, these results are completed and generalized in Part B, which describes more generally the irreducible representations of a finite group with a split BN-pair. This part also contains a survey of properties of finite groups with BN-pairs.

Parts C and D are devoted to representations over the complex numbers. C gives an account of Harish-Chandra's adaptation of Langlands' work on Eisenstein series to the representation theory of

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the groups of rational points of a reductive group defined over a finite field k. Part D discusses, as much as possible from the point of view of the general theory, the special cases known so far, namely the results of Green on $\underline{\underline{GL}}_n(k)$, of Schur on $\underline{\underline{SL}}_2(k)$ and of B. Srinivasan on $\underline{\underline{Sp}}_4(k)$.

Part E gives an extensive survey of results, partly with proofs, and of problems on conjugacy classes in semi-simple algebraic groups and in their Lie algebras. Part F describes explicitly the classes of involutions, and the corresponding centralizers for all the Chevalley groups of simple type, in particular over finite fields. Finally, Part G outlines an approach to the determination of the conjugacy classes in Weyl groups of simple algebraic groups, and gives a complete enumeration of them for each type.

A. Borel

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