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Realization Theory and Design of Digital Images

With 49 Figures

Springer

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Preface

This monograph is concerned with description and design for two-dimensional and three-dimensional images; it will be of special interest to researchers and graduate students who specialized in image processing and system theory. From the data in digital images, mathematical models will be constructed. Then new systems which describe faithfully any two-dimensional or three-dimensional digital images will be proposed. Using the systems thus allows description to be treated as realization problem and design. By virtue of this approach, this monograph provides new results and their extensions which are designing of two-dimensional and three-dimensional images. Some actual design examples will be also shown.

In usual image processing today, two-dimensional images are transformed into one-dimensional signals, then which are analyzed by means of various established methods in signal processing theory. Likewise, three-dimensional images are transformed into two-dimensional signals and these signals are analyzed by established methods in two-dimensional signal processing theory. Another common processing procedure employs tree structures such as quad-trees for two-dimensional images and oct-trees for three-dimensional ones.

In this monograph, two-dimensional and three-dimensional images are viewed as input/output relations with special features, which are special cases of the behavior in Linear Representation Systems as discussed in our *Realization Theory of Discrete-Time Dynamical Systems* (T. Matsuo and Y. Hasegawa, Lecture Notes in Control and Information Science, Vol. 296, Springer, 2003). The processing method which we present in this monograph is built on this new insight and is very well-adapted to input/output relations. Capturing special features in two-dimensional and three-dimensional images and extending the Linear Representations Systems discussed in the previous book, this monograph will present new results for image processing by transforming a description problem into a realization problem. It will show how to design images by computer, and it will be clear that anyone can easily design images by using new method.

Realization and design problems for digital images can be roughly stated as follows:

A. For a given digital image, finding a mathematical model, equivalently a dynamical system, with the same image.
B. If possible, clarifying when the mathematical model can be actually embodied. In other words, investigating whether the mathematical model is finite dimensional.
C. Determining the mathematical model from a finite-sized digital image. This problem is called a partial realization problem.
D. If possible, finding the simplest mathematical model among all the models which describe the image. This problem is called a structure problem.
E. If possible, constructing the mathematical model for an image which exists wholly in our mind. This problem is very common, but at least so far has not been able to be managed using computers.

It is good to remember that the development of signal processing, including filtering, has been strongly stimulated by linear system theory and well-connected with the related mathematics. However, more such development of image processing has not occurred yet because there have been no suitable mathematical models for images.

In this respect, we submit that our new findings and proposals for digital images, fully discussed in this monograph, are important.

Image processing may have become a theme of scientific technology after the facsimile was invented in about 1840. In those days, the goal was only to transmit an image from one place to another by wire or wireless. Hence a method for converting an image, i.e., a two-dimensional signal, into a one-dimensional signal for instant transmission was developed. However, analysis of the image itself was never undertaken.

As already said, the usual method of treating digital images, whether two- or three-dimensional ones is almost the same as that for standard audio signal processing. There are also specialized methods such as one using quad-trees for two-dimensional images and oct-trees for three-dimensional images, or shape analysis. However, our method for processing two-dimensional or three dimensional images takes into account the image values which show the connections between horizontal and vertical and depth positions of the image's components.

Recently, computer graphics has been used to portray realities of natural objects or phenomena. Such portrayal has grown into an art genre called computer graphic arts and explosive works are shown in many museums. We also remember that such process of developments have originated in the pictorial art. Usually, generation algorithms for a work of graphic art are irregular and complex. On the contrary, design for traditional or folk handicraft articles such as furniture decoration or clothing are regular and simple. Such patterns may be more pleasing to human beings. This monograph treats geometrical patterns of the folk design type. They are generated by mathematical models which are suitable for computer programs. Our design problems are especially intended for regular patterns, that is, periodical designs.

It is also noteworthy that our method intensionally takes a positive attitude toward using computers. We will introduce a new system called a Commutative Linear Representation System which will realize, that is faithfully describe, any image.

We wish to acknowledge Professor Tsuyoshi Matsuo, who established the foundation for realization theory of two-dimensional images, and who taught us much on realization theory for discrete-time non-linear systems. He would be an author of this monograph, but in April thirteen years ago be sadly passed away. We greatfully consider him one of the authors of this manuscript in spirit.

We also wish to thank Professor R. E. Kalman for his suggestions. He stimulated us to research these realization problems directly as well as through his works.

Finally, Professor Kazuyasu Hamada pointed out some minor errors.

We also thank Special Lecturer M. L. Roecklein for making the first manuscript into a more elegant one.

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