

Lecture Notes
in Control and Information Sciences

342

Editors: M. Thoma · M. Morari

Yasumichi Hasegawa · Tatsuo Suzuki

Realization Theory and Design of Digital Images

With 49 Figures

 Springer

Series Advisory Board

F. Allgöwer · P. Fleming · P. Kokotovic · A.B. Kurzhanski ·
H. Kwakernaak · A. Rantzer · J.N. Tsitsiklis

Authors

Prof. Yasumichi Hasegawa

Dr. Tatsuo Suzuki

Gifu University

Department of Electronics and Computer Engineering

Yanagido 1-1

501-11 Gifu

Japan

yhasega@gifu-u.ac.jp

tasuzuki@gifu-u.ac.jp

ISSN 0170-8643

ISBN-10 3-540-36115-4 **Springer Berlin Heidelberg New York**

ISBN-13 978-3-540-36115-2 **Springer Berlin Heidelberg New York**

Library of Congress Control Number: 2006928833

This work is subject to copyright. All rights are reserved, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilm or in other ways, and storage in data banks. Duplication of this publication or parts thereof is permitted only under the provisions of the German Copyright Law of September 9, 1965, in its current version, and permission for use must always be obtained from Springer. Violations are liable to prosecution under German Copyright Law.

Springer is a part of Springer Science+Business Media

springer.com

© Springer-Verlag Berlin Heidelberg 2006

Printed in Germany

The use of general descriptive names, registered names, trademarks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

Typesetting: Data conversion by authors.

Final processing by PTP-Berlin Protago- \TeX -Production GmbH, Germany (www.ptp-berlin.com)

Cover-Design: WMXDesign, Heidelberg

Printed on acid-free paper 89/3141/Yu - 5 4 3 2 1 0

Preface

This monograph is concerned with description and design for two-dimensional and three-dimensional images; it will be of special interest to researchers and graduate students who specialized in image processing and system theory. From the data in digital images, mathematical models will be constructed. Then new systems which describe faithfully any two-dimensional or three-dimensional digital images will be proposed. Using the systems thus allows description to be treated as realization problem and design. By virtue of this approach, this monograph provides new results and their extensions which are designing of two-dimensional and three-dimensional images. Some actual design examples will be also shown.

In usual image processing today, two-dimensional images are transformed into one-dimensional signals, then which are analyzed by means of various established methods in signal processing theory. Likewise, three-dimensional images are transformed into two-dimensional signals and these signals are analyzed by established methods in two-dimensional signal processing theory. Another common processing procedure employs tree structures such as quad-trees for two-dimensional images and oct-trees for three-dimensional ones.

In this monograph, two-dimensional and three-dimensional images are viewed as input/output relations with special features, which are special cases of the behavior in Linear Representation Systems as discussed in our *Realization Theory of Discrete-Time Dynamical Systems* (T. Matsuo and Y. Hasegawa, Lecture Notes in Control and Information Science, Vol. 296, Springer, 2003). The processing method which we present in this monograph is built on this new insight and is very well-adapted to input/output relations. Capturing special features in two-dimensional and three-dimensional images and extending the Linear Representations Systems discussed in the previous book, this monograph will present new results for image processing by transforming a description problem into a realization problem. It will show how to design images by computer, and it will be clear that anyone can easily design images by using new method.

Realization and design problems for digital images can be roughly stated as follows:

- A. For a given digital image, finding a mathematical model, equivalently a dynamical system, with the same image.
- B. If possible, clarifying when the mathematical model can be actually embodied. In other words, investigating whether the mathematical model is finite dimensional.
- C. Determining the mathematical model from a finite-sized digital image. This problem is called a partial realization problem.
- D. If possible, finding the simplest mathematical model among all the models which describe the image. This problem is called a structure problem.
- E. If possible, constructing the mathematical model for an image which exists wholly in our mind. This problem is very common, but at least so far has not been able to be managed using computers.

It is good to remember that the development of signal processing, including filtering, has been strongly stimulated by linear system theory and well-connected with the related mathematics. However, more such development of image processing has not occurred yet because there have been no suitable mathematical models for images.

In this respect, we submit that our new findings and proposals for digital images, fully discussed in this monograph, are important.

Image processing may have become a theme of scientific technology after the facsimile was invented in about 1840. In those days, the goal was only to transmit an image from one place to another by wire or wireless. Hence a method for converting an image, i.e., a two-dimensional signal, into a one-dimensional signal for instant transmission was developed. However, analysis of the image itself was never undertaken.

As already said, the usual method of treating digital images, whether two- or three-dimensional ones is almost the same as that for standard audio signal processing. There are also specialized methods such as one using quad-trees for two-dimensional images and oct-trees for three-dimensional images, or shape analysis. However, our method for processing two-dimensional or three dimensional images takes into account the image values which show the connections between horizontal and vertical and depth positions of the image's components.

Recently, computer graphics has been used to portray realities of natural objects or phenomena. Such portrayal has grown into an art genre called computer graphic arts and explosive works are shown in many museums. We also remember that such process of developments have originated in the pictorial art. Usually, generation algorithms for a work of graphic art are irregular and complex. On the contrary, design for traditional or folk handicraft articles such as furniture decoration or clothing are regular and simple. Such patterns may be more pleasing to human beings. This monograph treats geometrical patterns of the folk design type. They are generated by mathematical models which are suitable for computer programs. Our design problems are especially intended for regular patterns, that is, periodical designs.

It is also noteworthy that our method intensionally takes a positive attitude toward using computers. We will introduce a new system called a Commutative Linear Representation System which will realize, that is faithfully describe, any image.

We wish to acknowledge Professor Tsuyoshi Matsuo, who established the foundation for realization theory of two-dimensional images, and who taught us much on realization theory for discrete-time non-linear systems. He would be an author of this monograph, but in April thirteen years ago he sadly passed away. We greatly consider him one of the authors of this manuscript in spirit.

We also wish to thank Professor R. E. Kalman for his suggestions. He stimulated us to research these realization problems directly as well as through his works.

Finally, Professor Kazuyasu Hamada pointed out some minor errors.

We also thank Special Lecturer M.L. Roecklein for making the first manuscript into a more elegant one.

May 2006

Yasumichi Hasegawa
Tatsuo Suzuki

Contents

1	Introduction	1
2	Two-Dimensional Images and Three-Dimensional Images ..	9
2.1	Two-Dimensional Images and Input/Output Relations	9
2.2	Three-Dimensional Images	10
2.3	Historical Notes and Concluding Remarks	11
3	Realization Theory of Two-Dimensional Images	13
3.1	2-Commutative Linear Representation Systems	14
3.2	Definite Examples of Two-Dimensional Images Generated by Finite-Dimensional 2-Commutative Linear Representation Systems	18
3.3	Finite-Dimensional 2-Commutative Linear Representation Systems	19
3.4	Partial Realization Theory of Two-Dimensional Images	27
3.5	Historical Notes and Concluding Remarks	37
	Appendix to Chapter 3	39
3-A	Realization Theorem	39
3-A.1	Linear State Structure: $\{\alpha, \beta\}$ -Actions	39
3-A.2	Pointed $\{\alpha, \beta\}$ -Actions	43
3-A.3	$\{\alpha, \beta\}$ -Actions with a Readout Map	45
3-A.4	2-Commutative Linear Representation Systems	46
3-A.5	Sophisticated 2-Commutative Linear Representation System	47
3-B	Finite-Dimensional 2-Commutative Linear Representation Systems	49
3-B.1	Finite-Dimensional $\{\alpha, \beta\}$ -Actions and Pointed $\{\alpha, \beta\}$ -Actions	49
3-B.2	Finite-Dimensional $\{\alpha, \beta\}$ -Actions with a Readout Map	54
3-B.3	Finite-Dimensional 2-Commutative Linear Representation Systems	57

3-B.4	Existence Criterion for 2-Commutative Linear Representation Systems	58
3-B.5	Realization Procedure for 2-Commutative Linear Representation Systems	60
3-C	Partial Realization Theory	60
3-C.1	Pointed $\{\alpha, \beta\}$ -Actions	60
3-C.2	$\{\alpha, \beta\}$ -Actions with a Readout Map	61
3-C.3	Partial Realization Problem	62
4	Structures of 2-Commutative Linear Representation Systems	71
4.1	Structure Theory of 2-Commutative Linear Representation Systems	72
4.2	Structure Theory and a Coding Theory of Two-Dimensional Images	74
4.3	Historical Notes and Concluding Remarks	79
	Appendix to Chapter 4	80
5	Design for Two-Dimensional Images	89
5.1	2-Commutative Linear Representation Systems for Design	90
5.2	Design Methods for Geometrical Patterns	99
5.3	Historical Notes and Concluding Remarks	100
6	Realization Theory of Three-Dimensional Images	101
6.1	3-Commutative Linear Representation Systems	102
6.2	Definite Examples of Images Generated by Finite-Dimensional 3-Commutative Linear Representation Systems	104
6.3	Finite-Dimensional 3-Commutative Linear Representation Systems	107
6.4	Partial Realization of Three-Dimensional Images	117
6.5	Historical Notes and Concluding Remarks	130
	Appendix to Chapter 6	131
6-A	Proof of the Realization Theory of Three-Dimensional Images	131
6-A.1	Linear State Structure: $\{\alpha, \beta, \gamma\}$ -Actions	131
6-A.2	Pointed $\{\alpha, \beta, \gamma\}$ -Actions	133
6-A.3	$\{\alpha, \beta, \gamma\}$ -Actions with a Readout Map	134
6-A.4	3-Commutative Linear Representation System	136
6-A.5	Sophisticated 3-Commutative Linear Representation System	137
6-B	Finite-Dimensional 3-Commutative Linear Representation Systems	139
6-B.1	Finite-Dimensional $\{\alpha, \beta, \gamma\}$ -Actions	139

6-B.2	Finite-Dimensional Pointed $\{\alpha, \beta, \gamma\}$ -Actions	140
6-B.3	Finite-Dimensional $\{\alpha, \beta, \gamma\}$ -Actions with a Readout Map	144
6-B.4	Finite-Dimensional 3-Commutative Linear Representation Systems	145
6-B.5	Existence Criterion for Finite-Dimensional 3-Commutative Linear Representation Systems	145
6-B.6	Realization Procedure for Finite-Dimensional 3-Commutative Linear Representation Systems	147
6-C	Partial Realization Theory	147
6-C.1	Pointed $\{\alpha, \beta, \gamma\}$ -Actions	147
6-C.2	$\{\alpha, \beta, \gamma\}$ -Actions with a Readout Map	148
6-C.3	Partial Realization Problem	150
7	Structures of 3-Commutative Linear Representation Systems	159
7.1	Structure Theory of 3-Commutative Linear Representation Systems	160
7.2	Structure Theory and a Coding Theory of Three-Dimensional Images	163
7.3	Historical Notes and Concluding Remarks	176
	Appendix to Chapter 7	177
8	Design for Three-Dimensional Images	201
8.1	3-Commutative Linear Representation Systems for Design	202
8.2	Design Methods for Geometrical Patterns	213
8.3	Historical Notes and Concluding Remarks	214
	References	215
	Index	223