

Lecture Notes in Mathematics

An informal series of special lectures, seminars and reports on mathematical topics

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Introduction to Lie Groups and Transformation Groups

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P R E F A C E

These notes were written for introductory lectures on Lie groups and transformation groups, held at the Universities of Buenos Aires and Zurich. The notions of a differentiable manifold, a differentiable map and a vectorfield are supposed known. There is an appendix on categories and functors.

The first two chapters are influenced by a paper of R. Palais [12]. In sections 5.2 and 5.3, a lot is taken from S. Kobayashi and K. Nomizu [11]. In chapter 7, S. Helgason [6] was often used. Of course, C. Chevalley [3] was constantly consulted. The bibliography orients on the various sources. A special feature of this presentation is the systematic avoidance of the use of local coordinates on a manifold. This allows the use of the presented theory with slight modifications for Lie groups over Banach manifolds. See e. g. B. Maissen [10].

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