

Mikhail Itskov

Tensor Algebra and Tensor Analysis for Engineers

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With Applications to Continuum Mechanics

With 13 Figures and 3 Tables

 Springer

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Library of Congress Control Number: 2007920572

ISBN 978-3-540-36046-9 Springer Berlin Heidelberg New York

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Typesetting: camera-ready by the Author
Production: LE-TeX Jelonek, Schmidt & Vöckler GbR, Leipzig
Cover: eStudio Calamar, Spain

Printed on acid-free paper 7/3100YL - 5 4 3 2 1 0

Моим родителям

Preface

Like many other textbooks the present one is based on a lecture course given by the author for master students of the RWTH Aachen University. In spite of a somewhat difficult matter those students were able to endure and, as far as I know, are still fine. I wish the same for the reader of the book.

Although the present book can be referred to as a textbook one finds only little plain text inside. I tried to explain the matter in a brief way, nevertheless going into detail where necessary. I also avoided tedious introductions and lengthy remarks about the significance of one topic or another. A reader interested in tensor algebra and tensor analysis but preferring, however, words instead of equations can close this book immediately after having read the preface.

The reader is assumed to be familiar with the basics of matrix algebra and continuum mechanics and is encouraged to solve at least some of numerous exercises accompanying every chapter. Having read many other texts on mathematics and mechanics I was always upset vainly looking for solutions to the exercises which seemed to be most interesting for me. For this reason, all the exercises here are supplied with solutions amounting a substantial part of the book. Without doubt, this part facilitates a deeper understanding of the subject.

As a research work this book is open for discussion which will certainly contribute to improving the text for further editions. In this sense, I am very grateful for comments, suggestions and constructive criticism from the reader. I already expect such criticism for example with respect to the list of references which might be far from being complete. Indeed, throughout the book I only quote the sources indispensable to follow the exposition and notation. For this reason, I apologize to colleagues whose valuable contributions to the matter are not cited.

Finally, a word of acknowledgment is appropriate. I would like to thank Uwe Navrath for having prepared most of the figures for the book. Further, I am grateful to Alexander Ehret who taught me first steps as well as some “dirty” tricks in \LaTeX , which were absolutely necessary to bring the

manuscript to a printable form. He and Tran Dinh Tuyen are also acknowledged for careful proof reading and critical comments to an earlier version of the book. My special thanks go to the Springer-Verlag and in particular to Eva Hestermann-Beyerle and Monika Lempe for their friendly support in getting this book published.

Aachen, November 2006

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