Lecture Notes in Mathematics

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Distributions and Nonlinear Partial Differential Equations

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to my wife HERMONA

P R E F A C E

The nonlinear method in the theory of distributions presented in this work is based on embeddings of the distributions in $\mathcal{D}'(R^n)$ into associative and commutative algebras whose elements are classes of sequences of smooth functions on R^n. The embeddings define various distribution multiplications. Positive powers can also be defined for certain distributions, as for instance the Dirac δ function.

A framework is in that way obtained for the study of nonlinear partial differential equations with weak or distribution solutions as well as for a whole range of irregular operations on distributions, encountered for instance in quantum mechanics.

In chapter 1, the general method of constructing the algebras containing the distributions and basic properties of these algebras are presented. The way the algebras are constructed can be interpreted as a sequential completion of the space of smooth functions on R^n. In chapter 2, based on an analysis of classes of singularities of piece wise smooth functions on R^n, situated on arbitrary closed subsets of R^n with smooth boundaries, for instance, locally finite families of smooth surfaces, the so called Dirac algebras, which prove to be useful in later applications are introduced.

Chapter 3 presents a first application. A general class of nonlinear partial differential equations, with polynomial nonlinearities is considered. These equations include among others, the nonlinear hyperbolic equations modelling the shock waves as well as well known second order nonlinear wave equations. It is shown that the piece wise smooth weak solutions of the general nonlinear equations considered, satisfy the equations in the usual algebraic sense, with the multiplication and derivatives in the algebras containing the distributions. It follows in particular that the same holds for the piece wise smooth shock wave solutions of nonlinear hyperbolic equations.

A second application is given in chapter 4, where one and three dimensional quantum particle motions in potentials arbitrary positive powers of the Dirac δ function are considered. These potentials which are no more measures, present the strongest local singularities studied in scattering theory. It is proved that the wave function solutions obtained within the algebras containing the distributions, possess the scattering property of being solutions of the potential free equations on either side of the potentials while satisfying special junction relations on the support of the potentials. In chapter 5, relations involving irregular products with Dirac distributions are proved to be valid within the algebras containing the distributions. In particular, several known relations in quantum mechanics, involving irregular products with

Dirac and Heisenberg distributions are valid within the algebras. Chapter 6 presents the peculiar effect coordinate scaling has on Dirac distribution derivatives. That effect is a consequence of the condition of strong local presence the representations of the Dirac distribution satisfy in certain algebras. In chapter 7, local properties in the algebras are presented with the help of the notion of support, the local character of the product being one of the important results. Chapter 8 approaches the problem of vanishing and local vanishing of the sequences of smooth functions which generate the ideals used in the quotient construction giving the algebras containing the distributions. That problem proves to be closely connected with the necessary structure of the distribution multiplications. The method of sequential completion used in the construction of the algebras containing the distributions establishes a connection between the nonlinear theory of distributions presented in this work and the theory of algebras of continuous functions.

The present work resulted from an interest in the subject over the last few years and it was accomplished while the author was a member of the Applied Mathematics Group within the Department of Computer Science at Haifa Technion. In this respect, the author is particularly glad to express his special gratitude to Prof. A. Paz, the head of the department, for the kind support and understanding offered during the last years.

Many thanks go to the colleagues at Technion, M. Israeli and L. Shulman, for valuable reference indications, respectively for suggesting the scattering problem in potentials positive powers of the Dirac δ function, solved in chapter 4.

The author is indebted to Prof. B. Fuchssteiner from Paderborn, for his suggestions in contacting persons with the same research interest.

Lately, the author has learnt about a series of extensive papers of K. Keller, from the Institute for Theoretical Physics at Aachen, presenting a rather complementary approach to the problem of irregular operations with distributions. The author is very glad to thank him for the kind and thorough exchange of views.

A special gratitude and acknowledgement is expressed by the author to R.C. King from Southampton University, for his generosity in promptly offering the result on generalized Vandermonde determinants which corrects an earlier conjecture of the author and upon which the chapters 5 and 6 are based.

All the highly careful and demanding work of editing the manuscript was done by my wife Hermona, who inspite and on the account of her other much more interesting and elevated usual occupations found it necessary to support an effort in regularizing

the irregulars ..., in multiplying the distributions ...

By the way of multiplication: Prof. A. Ben-Israel, a former colleague, noticing the series of preprints, papers, etc. resulted from the author's interest in the subject and seemingly inspired by one of the basic commandments in the Bible, once quipped: "Be fruitful and multiply ... distributions ..."

E. E. R.

Haifa, December 1977

CONTENT

NOTE

The Reader interested mainly in NONLINEAR PARTIAL DIFFERENTIAL EQUATIONS, may at a first lecture concentrate on the following sections:

"Never forget

the beaches of ASHQELON ... "