

# Picture Processing and Digital Filtering

Edited by T.S. Huang

With Contributions by

H. C. Andrews F. C. Billingsley

J. G. Fiasconaro B. R. Frieden T. S. Huang

R. R. Read J. L. Shanks S. Treitel

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Professor THOMAS S. HUANG, PhD

Purdue University, School of Electrical Engineering  
West Lafayette, IN 47907, USA

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## **Preface to the Second Edition**

The warm reception accorded to our original volume made us decide to put out this updated paperback edition so that the book can be more accessible to graduate students.

This paperback edition is essentially identical to the original hard-cover one except for the addition of a new chapter (Chapter 7) which reviews the recent advances in two-dimensional transforms and filters, and image restoration and enhancement. One hundred additional references have been evaluated and cited. A few typographic errors in the original edition were corrected.

Because of limitation of space, we can do little more in Chapter 7 than point the reader to the literature. We hope that in-depth treatments of some of the important recent results in picture processing and digital filtering will appear in future volumes of the Springer physics program.

West Lafayette, Indiana

December 1978

THOMAS S. HUANG

## Preface to the First Edition

In every scientific and engineering endeavor, we encounter signal processing. Many signals are multi-dimensional, i. e., they are functions of several variables. Examples include medical and industrial radiographs, electron-micrographs, radar and sonar maps, seismic data, television images, and satellite (such as ERTS) photographs. The purpose of processing could be: signal generation and display, quality enhancement, information extraction, pattern recognition, efficient coding for transmission or storage, etc. Some of the useful and exciting applications of multi-dimensional signal processing are: character recognition, enhancement of satellite pictures of the moon and Mars, mapping of earth resources from ERTS photographs, and transaxial tomography.

Signal processing can be done either digitally or analogly. However, digital techniques are by far the more flexible. It is because of the rapid progress in digital technology that many multi-dimensional signal processing tasks have become feasible.

In the present book, we bring to the reader in-depth treatment of selected topics in the digital processing of two-dimensional signals (i. e., pictures or images): Chapters 2-4 are on two-dimensional transforms and filters, Chapter 5 is on image enhancement and restoration, and Chapter 6 is on the noise problem in digital signal processing hardware, especially scanners. These chapters are tutorial in nature, yet they bring the reader to the very forefront of current research. We envision that this book will be useful either as a reference book for working scientists and engineers or a supplementary text book in courses on digital signal processing, image processing, and digital filtering.

We are grateful to ARPA for supporting the editing of this book as well as the work described in the introductory chapter (under contract no. MDA 703-74-0098). We would also like to thank Professor Dr. ADOLF LOHMANN, Technical University of Erlangen, who first suggested the need for a book such as this.

West Lafayette, Indiana  
April 1975

THOMAS S. HUANG

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## Contributors

ANDREWS, HARRY C.

Department of Electrical Engineering and Computer Sciences Program,  
Image Processing Institute, University of Southern California,  
Los Angeles, CA 90007, USA

BILLINGSLEY, FREDERIC C.

Jet Propulsion Laboratory, California Institute of Technology,  
Pasadena, CA 91103, USA

FIASCONARO, JAMES G.

M. I. T. Lincoln Laboratory,  
Lexington, MA 02173, USA

FRIEDEN, B. ROY

Optical Sciences Center, University of Arizona,  
Tucson, AZ 85721, USA

HUANG, THOMAS S.

Purdue University, School of Electrical Engineering  
West Lafayette, IN 47907, USA

READ, RANDOL R.

SHANKS, JOHN L.

TREITEL, SVEN

Amoco Production Company, Research Center,  
Tulsa, OK 74102, USA