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Schwartz Spaces, Nuclear Spaces and Tensor Products

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TO MY WIFE

Judy, Dick-Ha Nei

INTRODUCTION

Nuclear spaces were discovered by A. Grothendieck, and the most important part of the theory of nuclear spaces was developed in his famous article (see Grothendieck [2]). Unfortunately, the machinery of tensor products used there is very cumbersome; this makes the theory some unnecessary difficulties and complicacy. Pietsch [1] was the first to simplify the theory of nuclear spaces by using locally convex spaces of summable and absolutely summable families, instead of locally convex tensor products. It is my impression that the idea of using seminorms satisfying some expected properties to investigate some classes of locally convex spaces, would make the development of the whole theory easier. This has been done by the author [1] for studying the topology σ_s of uniform convergence on order-bounded sets. Thus, one of the purposes of these lecture notes is to embark upon a unifying treatment of Schwartz spaces, nuclear spaces, and λ-nuclear spaces by using this idea. Therefore, this monograph is actually a continuation of that written by the author [1].

These lecture notes contain six chapters, starting with Chapter 0 in which terminology and some elementary results in topological vector spaces, that will be used through these notes, are given. Chapter 1 deals with Schwartz spaces by means of precompactness of seminorms. Chapter 3 is concerned with nuclear spaces in terms of prenuclearity of seminorms; in order to show that our definition of nuclearity coincides with the usual one, we use the concept of summability which is studied in the first two sections of Chapter 2.

In Chapter 4, the s-norm on a tensor product of normed spaces, studied by Cohen [1], is constructed in a natural way by asking when its Banach dual is isometrically isomorphic to the well-known Banach space of all absolutely summing mappings equipped with the absolutely summing norm. Using the same idea we construct the ℓ-norm on a tensor product of ordered normed spaces from the well-known Banach space of all cone-absolutely summing mappings equipped with a suitable norm. The ℓ-norm has been studied by Schaefer [2] for the tensor product of Banach lattices. Also in Chapter 5, we study two locally solid

topologies on the tensor product, equipped with the projective and biprojective cone, of two locally solid spaces.

A few words should be said about the reference system in these notes, (i, j, k) will denote the k-th proposition or theorem in Chapter i, Section j.

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Y.C.W.

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