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*-Autonomous Categories

With an Appendix by Po-Hsiang Chu

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PREFACE

The category of finite dimensional vector spaces over a field K has many interesting properties: It is a symmetric closed monoidal (hereafter known as autonomous) category which has an object K, with the property that the functor (-,K), internal Hom into K, induces an equivalence with its opposite category. Similar remarks apply to the category of finite dimensional (real or complex) banach spaces. We call such a category *-autonomous. Almost the same thing happens with finite abelian groups, except the "dualizing object", $\mathbb{R}/\mathbb{Z}$ or $\mathbb{Q}/\mathbb{Z}$, is not an object of the category. In no case is the category involved complete, nor is there an obvious way of extending both the closed structure and the duality to any of the completions. In studying these phenomena, I came on a fairly general construction which allows you to begin with one of the above categories (and some similar ones) to embed it fully into a complete and cocomplete category which admits an autonomous structure and which, using the original dualizing object, is *-autonomous.

In an appendix, my student Po-Hsiang Chu describes a construction which embeds any autonomous category into a *-autonomous category. The embedding described is not, however, full and is completely formal.

The work described here was carried out during a sabbatical leave from McGill University, academic year 1975-76 mostly at the Forschungsinstitut für Mathematik der Eidgenossische Technische Hochschule, Zürich. For shorter periods I was at Universitetet i Aarhus as well as l'Universite Catholique de Louvain (Louvain-la-Neuve) and I would like to thank all these institutions. I was partially supported during that year by a leave fellowship from the Canada Council and received research grants from the National Research Council and the Ministère de l'Education du Québec.

Preliminary versions of part of this material has been published in the five papers by me listed in the bibliography. The current version was presented in a series of lectures at McGill in the Winter Term, 1976.

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APPENDIX

CONSTRUCTING *-AUTONOMOUS CATEGORIES

Po-Hsiang Chu