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Editors

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Domain Decomposition Methods in Science and Engineering XVI

With 222 Figures and 99 Tables

Springer

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Preface

This volume is the definitive technical record of advances in the analysis algorithmic development, large-scale implementation, and application of domain decomposition methods in science and engineering presented at the Sixteenth International Conference on Domain Decomposition Methods. The conference was held in New York City, January 11-15, 2005. The largest meeting in this series to date, it registered 228 participants from 20 countries. The Courant Institute of Mathematical Sciences of New York University hosted the technical sessions. The School of Engineering and Applied Science of Columbia University hosted a pre-conference workshop on software for domain decomposition methods.

1 Background of the Conference Series

The International Conference on Domain Decomposition Methods has been held in eleven countries throughout Asia, Europe, and North America, beginning in Paris in 1987. Originally held annually, it is now spaced out at roughly 18-month intervals. A complete list of past meetings appears below.

The sixteenth instance of the International Conference on Domain Decomposition Methods was the sixth in the United States, and the first since 1997. In 1997, ASCI Red, the world's first Teraflops-scale computer, was just being placed into service at Sandia National Laboratories. The Bell Prize was won by an application that sustained 170 Gflop/s that year. An entirely new fleet of machines, algorithms, and codes has swept the research community in the intervening years. Now the Top 500 supercomputers in the world all sustain 2.0 Teraflop/s or more on the ScaLAPACK benchmark and nearly 200 Tflop/s have been sustained in simulations submitted to the Bell Prize competition.

The principal technical content of the conference has always been mathematical, but the principal motivation has been to make efficient use of distributed memory computers for complex applications arising in science and engineering. Thus, contributions from mathematicians, computer scientists, engineers, and scientists have always been welcome. Though the conference has grown up in the wake of commercial massively parallel processors, it is worth noting that many interesting applications of domain decomposition are not massively parallel at all. "Gluing together" just two subproblems to effectively exploit a different solver on each is also part of the technical fabric of

the conference. Even as multiprocessing becomes commonplace, multiphysics modeling is in ascendancy, so the International Conference on Domain Decomposition Methods remains as relevant and as fundamentally interdisciplinary as ever. While research in domain decomposition methods is presented at numerous venues, the International Conference on Domain Decomposition Methods is the only regularly occurring international forum dedicated to interdisciplinary technical interactions between theoreticians and practitioners working in the creation, analysis, software implementation, and application of domain decomposition methods.

International Conferences on Domain Decomposition Methods:

- Paris, France, 1987
- Los Angeles, USA, 1988
- Houston, USA, 1989
- Moscow, USSR, 1990
- Norfolk, USA, 1991
- Como, Italy, 1992
- University Park (Pennsylvania), USA, 1993
- Beijing, China, 1995
- Ullensvang, Norway, 1996
- Boulder, USA, 1997
- Greenwich, UK, 1998
- Chiba, Japan, 1999
- Lyon, France, 2000
- Cocoyoc, Mexico, 2002
- Berlin, Germany, 2003
- New York, USA, 2005

International Scientific Committee on Domain Decomposition Methods:

- Petter Bjørstad, Bergen
- Roland Glowinski, Houston
- Ronald Hoppe, Augsburg & Houston
- Hideo Kawarada, Chiba
- David Keyes, New York
- Ralf Kornhuber, Berlin
- Yuri Kuznetsov, Houston
- Ulrich Langer, Linz
- Jacques Périaux, Paris
- Olivier Pironneau, Paris
- Alfio Quarteroni, Lausanne
- Zhong-ci Shi, Beijing
- Olof Widlund, New York
- Jinchao Xu, University Park

2 About the Sixteenth Conference

The 3.5-day conference featured 14 invited speakers, who were selected from about three times this number of nominees by the International Scientific Committee, with the goals of mixing traditional leaders and "new blood," featuring mainstream and new directions, and reflecting the international diversity of the community. There were 160 presentations altogether. Sponsorship from several U.S. scientific agencies and organizations (listed below) made it possible to offer about 20 travel fellowships to graduate students and postdocs from the U.S. and abroad.

Sponsoring Organizations:

- Argonne National Laboratory
- Lawrence Livermore National Laboratory
- Sandia National Laboratories
- U. S. Army Research Office
- U. S. Department of Energy, National Nuclear Security Administration
- U. S. National Science Foundation
- U. S. Office of Naval Research

Cooperating Organizations:

- Columbia University, School of Engineering & Applied Sciences
- New York University, Courant Institute of Mathematical Sciences
- Society for Industrial and Applied Mathematics, Activity Group on Supercomputing

Local Organizing Committee Members:

- Randolph E. Bank, University of California, San Diego
- Timothy J. Barth, NASA Ames Research Center
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- Marc Spiegelman, Columbia University
- Ray Tuminaro, Sandia National Laboratory
- Panayot Vassilevski, Lawrence Livermore National Laboratory
- Olof Widlund, New York University (Co-Chair)
- Margaret H. Wright, New York University

3 About Domain Decomposition Methods

Domain decomposition, a form of divide-and-conquer for mathematical problems posed over a physical domain, as in partial differential equations, is the most common paradigm for large-scale simulation on massively parallel, distributed, hierarchical memory computers. In domain decomposition, a large problem is reduced to a collection of smaller problems, each of which is easier to solve computationally than the undecomposed problem, and most or all of which can be solved independently and concurrently. Typically, it is necessary to iterate over the collection of smaller problems, and much of the theoretical interest in domain decomposition algorithms lies in ensuring that the number of iterations required is very small. Indeed, the best domain decomposition methods share with their cousins, multigrid methods, the property that the total computational work is linearly proportional to the size of the input data, or that the number of iterations required is at most logarithmic in the number of degrees of freedom of individual subdomains.

Algorithms whose work requirements are linear in the size of the input data in this context are said to be "optimal." Near optimal domain decomposition algorithms are now known for many, but certainly not all, important classes of problems that arise science and engineering. Much of the contemporary interest in domain decomposition algorithms lies in extending the classes of problems for which optimal algorithms are known.

Domain decomposition algorithms can be tailored to the properties of the physical system as reflected in the mathematical operators, to the number of processors available, and even to specific architectural parameters, such as cache size and the ratio of memory bandwidth to floating point processing rate.

Domain decomposition has proved to be an ideal paradigm not only for execution on advanced architecture computers, but also for the development of reusable, portable software. The most complex operation in a typical domain decomposition method — the application of the preconditioner — carries out in each subdomain steps nearly identical to those required to apply a conventional preconditioner to the undecomposed domain. Hence software developed for the global problem can readily be adapted to the local problem, instantly presenting lots of "legacy" scientific code for to be harvested for parallel implementations. Furthermore, since the majority of data sharing between subdomains in domain decomposition codes occurs in two archetypal communication operations — ghost point updates in overlapping zones between neighboring subdomains, and global reduction operations, as in forming an inner product — domain decomposition methods map readily onto optimized, standardized message-passing environments, such as MPI.

Finally, it should be noted that domain decomposition is often a natural paradigm for the modeling community. Physical systems are often decomposed into two or more contiguous subdomains based on phenomenological considerations, such as the importance or negligibility of viscosity or reactivity, or

any other feature, and the subdomains are discretized accordingly, as independent tasks. This physically-based domain decomposition may be mirrored in the software engineering of the corresponding code, and leads to threads of execution that operate on contiguous subdomain blocks. These can be either further subdivided or aggregated to fit the granularity of an available parallel computer.

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5 Note Concerning Abstracts and Presentations

Within each section of plenary, minisymposium, and contributed papers, the edited proceedings appear in alphabetical order by first-listed author.

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Contents