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Large Coulomb Systems

Lecture Notes on Mathematical Aspects of QED

Springer

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Preface

Since the discovery of its fundamentals by Heisenberg, Born, Jordan, Schrödinger, and Dirac, quantum mechanics turned out to be one of the most successful theories of modern science. One of its cornerstones, the many-body Schrödinger equation with Coulomb interactions, was applied successfully to explain various properties of matter, such as the properties of light atoms.

In addition to its triumph as a physical theory, quantum mechanics has been mathematically formulated and many of its most basic statements have been proven, e.g., the self-adjointness of atomic and molecular Hamiltonians, asymptotic completeness, and the stability of matter. Various effective one-particle models – such as the Thomas-Fermi and the Hartree-Fock theory – were shown to be limiting models of fully interacting Coulomb systems.

Soon after its discovery, however, physicists realized that it is difficult to formulate a consistent and relativistic theory of matter interacting with the electromagnetic field. These difficulties have been skillfully treated by the great masters of physics belonging to the middle of the last century, e.g., Feynman, Dyson, Schwinger and Tomonaga. In fact, quantum electrodynamics (QED), the theory developed by them, has an admirable quantitative predictive power.

Nevertheless, this contrasts with the mathematical status of QED: its consistent mathematical formulation is unknown. In fact, not even a generally acknowledged multiparticle theory of relativistic electrons is available.

In recent years a renewed interest in these fundamental questions arose. Even if a fully satisfactory mathematical formulation of QED is so far unavailable, various effective models based on QED are within the reach of mathematical methods and generate numerous new and interesting results.

One of the activities in the field is the IHP-network "Analysis and Quantum" financed by the European Union. To introduce young postdocs into the field, it organized the summer school "Large Quantum Systems – QED" in Nordfjordeid, Norway, August 11–18, 2003. The present collection of reviews is based on the lecture notes of this school. In addition, one of the reviews is based on lectures from the summer school "Quantum Field Theory – from a Hamiltonian point of view" held in Sandbjerg Manor, Denmark, 2–9, August 2000, which was organized by the EU Network "PDE and Applications Quantum Mechanics".

We hope that this collection of reviews will be well received by all those interested in these developments.

Warsaw and Munich *Jan Dereziński*
November 2005 *Heinz Siedentop*

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