

Lecture Notes  
in Control and Information Sciences

317

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Editors: M. Thoma · M. Morari

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# **Nonblocking Supervisory Control of State Tree Structures**

With 114 Figures

 Springer

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# Preface

*Supervisory control* refers to control of discrete-event systems. In such a system the logical order of the events is of concern and not the time at which the events take place. Examples of engineering control problems for which supervisory control is useful include: control of heating and ventilation systems, failure diagnosis of communication networks, design of logical controllers in automated cars, control of a metro network, etc. The RW-framework developed by P.J. Ramadge and W.M. Wonham in terms of generators, directly related to automata, is now widely used. There exists now a body of theory on the existence of a supervisor which achieves control objectives of safety and required behavior, and for their computation. A bottleneck for the application of the theory to engineering examples is the large computational complexity of controllers. Approaches to lower the complexity are modular and hierarchical control. The existing algorithms for modular control in general still require a large complexity. Hence the interest in hierarchical control.

A formalism for hierarchical systems and for their model-checking was proposed by David Harel in a paper published in 1987. For control of discrete-event systems a similar approach was developed by W.M. Wonham and his Ph.D. students but the computational and complexity issues were still formidable. This research program is continued with this book. The plan of attack, constructed in hindsight, specified: (1) a new formalism for supervisory control of hierarchical systems; (2) specifications formulated in terms of logical formulas so as to lower their computational complexity; and (3) the use of algorithms based on binary decision diagrams (BDDs). This plan has been successfully carried out by Chuan Ma during his Ph.D. study under supervision of W. Murray Wonham. The results of this book constitute a major advance for supervisory control. Yet, more research and experience is required to make the theory and algorithms into an effective tool for control engineering of discrete-event systems.

The undersigned highly recommends the reading of this book to researchers in control of discrete-event systems. The authors are congratulated with the appearance of this book!

*Amsterdam, 24 January 2005*

*Jan H. van Schuppen*

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