Lecture Notes in Mathematics

Edited by J.-M. Morel, F. Takens and B. Teissier

Editorial Policy for Multi-Author Publications: Summer Schools / Intensive Courses

1. Lecture Notes aim to report new developments in all areas of mathematics and their applications – quickly, informally and at a high level. Mathematical texts analysing new developments in modelling and numerical simulation are welcome. Manuscripts should be reasonably self-contained and rounded off. Thus they may, and often will, present not only results of the author but also related work by other people. They should provide sufficient motivation, examples and applications. There should also be an introduction making the text comprehensible to a wider audience. This clearly distinguishes Lecture Notes from journal articles or technical reports which normally are very concise. Articles intended for a journal but too long to be accepted by most journals, usually do not have this "lecture notes" character.

2. In general SUMMER SCHOOLS and other similar INTENSIVE COURSES are held to present mathematical topics that are close to the frontiers of recent research to an audience at the beginning or intermediate graduate level, who may want to continue with this area of work, for a thesis or later. This makes demands on the didactic aspects of the presentation. Because the subjects of such schools are advanced, there often exists no textbook, and so ideally, the publication resulting from such a school could be a first approximation to such a textbook.
   Usually several authors are involved in the writing, so it is not always simple to obtain a unified approach to the presentation.
   For prospective publication in LNM, the resulting manuscript should not be just a collection of course notes, each of which has been developed by an individual author with little or no co-ordination with the others, and with little or no common concept. The subject matter should dictate the structure of the book, and the authorship of each part or chapter should take secondary importance. Of course the choice of authors is crucial to the quality of the material at the school and in the book, and the intention here is not to belittle their impact, but simply to say that the book should be planned to be written by these authors jointly, and not just assembled as a result of what these authors happen to submit.
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Metamorphoses of Hamiltonian Systems with Symmetries

Springer

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Preface

In these notes we apply modern methods of classical mechanics to the study of physical systems with symmetries, including, exact or approximate $\mathbf{S}^1 = \mathrm{SO}(2)$ (continuous) symmetries and discrete symmetries. In all cases the existence of a symmetry has profound implications for the dynamical behavior of such systems and for their basic qualitative properties. We are particularly interested in the following qualitative properties

- ▷ The existence and stability of relative equilibria, i.e. orbits of the system that are also group orbits of the $\mathbf{S}^1$ action.
- ▷ The behavior of periodic orbits near equilibria when the latter change stability, in particular, the Hamiltonian Hopf bifurcation.
- ▷ The topological properties of the foliation of the phase space by invariant tori in the case of completely integrable systems, in particular, monodromy.

Moreover, we are interested in how these basic qualitative features change as the parameters of these systems change, for example, we are interested in the bifurcations of periodic orbits or in the bifurcations of the topology of the integrable foliation of the phase space. I use the term *'metamorphosis'* in order to describe the *ensemble* of all such qualitative bifurcations that happen at certain values of the parameters and which affect the global qualitative picture of the dynamics[1].

We study four systems: the triply degenerate vibrational mode of tetrahedral molecules, the hydrogen atom in crossed electric and magnetic fields, a 'spherical pendulum' model of floppy molecules like LiCN and finally the $1:-2$ resonance which can serve as a local approximation of the dynamics near a resonant equilibrium.

As we go through these systems one by one, we see a number of important qualitative phenomena unfolding. In the triply degenerate vibrational mode of tetrahedral molecules we use the action of the tetrahedral group in order to

[1] The first word I thought of in order to describe this notion was the Russian 'perestroika'. I chose 'metamorphosis' after reading the preface of [10].

find the relative equilibria of the system and then we combine this study with Morse theory in the spirit of Smale [115, 116]. One of the families of relative equilibria in this system goes through a linear Hamiltonian Hopf bifurcation that is degenerate at the approximation used.

Hamiltonian Hopf bifurcations are studied in detail in the next two systems: the hydrogen atom in crossed fields and the family of spherical pendula. The main difference between the two systems with regards to the Hamiltonian Hopf bifurcation is that in the hydrogen atom the frequencies of the equilibrium that goes through the bifurcation collide on the imaginary axis and then move to the complex plane. On the other hand, in the family of spherical pendula we have a discrete (time-reversal) symmetry that forces the two frequencies of the equilibrium to be identical. In these two systems we study also the relation between the Hamiltonian Hopf bifurcations and the appearance of monodromy in the integrable foliation.

Ordinary monodromy can not be defined in the $1:-2$ resonance. A generalized notion of monodromy, which can be defined in the $1:-2$ resonance, was introduced in [99]. We describe this generalization, called *fractional monodromy*, in terms of period lattices and we sketch a proof.

I carried out this research as a PhD student at the Université du Littoral in Dunkerque with the support of the European Union Research Training Network MASIE. I would like to thank my supervisor Prof. Boris Zhilinskií of the Université du Littoral for his support during this work.

I am also very grateful to Dr. Dmitrií Sadovskií of the Université du Littoral and Dr. Richard Cushman of the Universiteit Utrecht for their advice and guidance during my PhD studies and for encouraging me to publish these notes. Some parts of this volume have been the result of our joint work and I would like to thank them for their kind permission to use here material from our papers [44] and [46].[2]

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[2] Parts of chapters 2 and 3 have appeared before in the papers [46] and [44] respectively.

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