Lecture Notes in Mathematics

Edited by J.-M. Morel, F. Takens and B. Teissier

Editorial Policy for Multi-Author Publications: Summer Schools / Intensive Courses

1. Lecture Notes aim to report new developments in all areas of mathematics and their applications – quickly, informally and at a high level. Mathematical texts analysing new developments in modelling and numerical simulation are welcome. Manuscripts should be reasonably self-contained and rounded off. Thus they may, and often will, present not only results of the author but also related work by other people. They should provide sufficient motivation, examples and applications. There should also be an introduction making the text comprehensible to a wider audience. This clearly distinguishes Lecture Notes from journal articles or technical reports which normally are very concise. Articles intended for a journal but too long to be accepted by most journals, usually do not have this "lecture notes" character.

2. In general SUMMER SCHOOLS and other similar INTENSIVE COURSES are held to present mathematical topics that are close to the frontiers of recent research to an audience at the beginning or intermediate graduate level, who may want to continue with this area of work, for a thesis or later. This makes demands on the didactic aspects of the presentation. Because the subjects of such schools are advanced, there often exists no textbook, and so ideally, the publication resulting from such a school could be a first approximation to such a textbook.
   Usually several authors are involved in the writing, so it is not always simple to obtain a unified approach to the presentation.
   For prospective publication in LNM, the resulting manuscript should not be just a collection of course notes, each of which has been developed by an individual author with little or no co-ordination with the others, and with little or no common concept. The subject matter should dictate the structure of the book, and the authorship of each part or chapter should take secondary importance. Of course the choice of authors is crucial to the quality of the material at the school and in the book, and the intention here is not to belittle their impact, but simply to say that the book should be planned to be written by these authors jointly, and not just assembled as a result of what these authors happen to submit.
   This represents considerable preparatory work (as it is imperative to ensure that the authors know these criteria before they invest work on a manuscript), and also considerable editing work afterwards, to get the book into final shape. Still it is the form that holds the most promise of a successful book that will be used by its intended audience, rather than yet another volume of proceedings for the library shelf.

3. Manuscripts should be submitted (preferably in duplicate) either to Springer's mathematics editorial in Heidelberg, or to one of the series editors (with a copy to Springer). Volume editors are expected to arrange for the refereeing, to the usual scientific standards, of the individual contributions. If the resulting reports can be forwarded to us (series editors or Springer) this is very helpful. If no reports are forwarded or if other questions remain unclear in respect of homogeneity etc, the series editors may wish to consult external referees for an overall evaluation of the volume. A final decision to publish can be made only on the basis of the complete manuscript, however a preliminary decision can be based on a pre-final or incomplete manuscript. The strict minimum amount of material that will be considered should include a detailed outline describing the planned contents of each chapter.
   Volume editors and authors should be aware that incomplete or insufficiently close to final manuscripts almost always result in longer evaluation times. They should also be aware that parallel submission of their manuscript to another publisher while under consideration for LNM will in general lead to immediate rejection.

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James Sneyd (Ed.)

Tutorials in Mathematical Biosciences II

Mathematical Modeling of Calcium Dynamics and Signal Transduction

With Contributions by:

R. Bertram · J.L. Greenstein · R. Hinch
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Preface

This is the second volume in the series "Lectures in Mathematical Biosciences". These lectures are based on material which was presented in tutorials or developed by visitors and postdoctoral fellows of the Mathematical Bioscience Institute (MBI), at The Ohio State University. The aim of this series is to introduce graduate students and researchers with just a little background in either mathematics or biology to mathematical modeling of biological processes. The present volume is devoted to Mathematical Modeling of Calcium Dynamics and Signal Transduction, which was the focus of the MBI program in the winter of 2004; documentation of that program, including streaming videos of the workshops, can be found on the website http://mbi.osu.edu.

This volume was organized and edited by James Sneyd. Sneyd is a world leader in mathematical physiology and biology, and has been working extensively on modeling biological processes of signal transduction induced by calcium oscillations.

Some of the chapters describe mathematical models of calcium dynamics as they occur in signal transduction. However, more attention is given in this volume to the underlying physiology, since, as Sneyd says, "Mathematical physiology is not possible without the physiology."

I wish to express my thanks to the contributors, all of whom served also as tutorial lecturers at the MBI. Special thanks are due to James Sneyd, who took it upon himself to organize this volume. I hope this volume will serve as a useful introduction to those who are interested in learning about mathematical physiology, and maybe even participating in this exciting field of research.

April, 2005 *Avner Friedman*

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Introduction

The question of how cells respond to their environment and coordinate their behavior with that of other cells is one that can naturally be studied using mathematical models. In order for cells to communicate with each other or with the outside world they have developed a large number of transduction mechanisms, whereby extracellular signals can be translated into intracellular signals, or a signal of one type can be changed into a signal of another type. For instance, muscle cells change an electrical signal into a force; photoreceptors change a light signal into an electrical signal; neurosecretory cells change an electrical signal into a hormonal signal; while in many cell types binding of a neurotransmitter or a hormone leads to oscillations in the concentration of intracellular free calcium, oscillations which control a variety of intracellular processes, including secretion, gene expression, cell movement, or wound repair. For instance, in muscle cells, the release of calcium from the sarcoplasmic reticulum controls muscle contraction, while in olfactory neurons and photoreceptors calcium forms an important negative feedback loop that controls adaptation. In neurosecretory cells, oscillations of the cytoplasmic calcium concentration lead to hormone secretion, while in neurons, calcium is not only crucial for synaptic communication, it is also an important modulator of synaptic plasticity.

In this volume we present a number of examples of signal transduction, showing how physiology and mathematical modeling interact to give a detailed quantitative understanding of such complex phenomena. Because of the widespread importance of calcium, all the chapters here will necessarily include discussion of calcium dynamics. Thus, we begin by showing how mathematical models of calcium dynamics can be constructed and analyzed. This is followed by descriptions of excitation-contraction coupling (i.e., how calcium forms a link between the muscle action potential and the contractile proteins), how the muscle proteins themselves work, and, finally, chapters on the physiology and modeling of olfactory neurons and of neuronal synapses.

Although this volume is part of a series of Tutorials in Mathematics, readers will soon notice that much of what is presented here is not mathematics at all, but physiology. This is entirely deliberate. It cannot be emphasized too strongly that mathematical physiology is not possible without the physiology. Without a detailed understanding of the physiology of the system under discussion it is simply not possible to say anything very interesting about it, no matter how clever is the mathematics used. Three of the authors here (Sanderson, Shannon and Reisert) are experimental physiologists, each of whom works closely with mathematical modelers to incorporate sophisticated modeling techniques into their research. The other four authors are primarily modelers, but ones that work closely with physiologists, or even, in some cases, do some experimental work themselves.

It is to be hoped that this combination of physiology and mathematics in what is, primarily, a volume for mathematicians, will be of use to all those who are interested in learning how modeling is done, or maybe even participating themselves in this most satisfying of endeavors.

April, 2005 *James Sneyd*