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Understanding and Using Linear Programming

 Springer

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Preface

This is an introductory textbook of linear programming, written mainly for students of computer science and mathematics. Our guiding phrase is, “what every theoretical computer scientist should know about linear programming.”

The book is relatively concise, in order to allow the reader to focus on the basic ideas. For a number of topics commonly appearing in thicker books on the subject, we were seriously tempted to add them to the main text, but we decided to present them only very briefly in a separate glossary. At the same time, we aim at covering the main results with complete proofs and in sufficient detail, in a way ready for presentation in class.

One of the main focuses is applications of linear programming, both in practice and in theory. Linear programming has become an extremely flexible tool in theoretical computer science and in mathematics. While many of the finest modern applications are much too complicated to be included in an introductory text, we hope to communicate some of the flavor (and excitement) of such applications on simpler examples.

We present three main computational methods. The simplex algorithm is first introduced on examples, and then we cover the general theory, putting less emphasis on implementation details. For the ellipsoid method we give the algorithm and the main claims required for its analysis, omitting some technical details. From the vast family of interior point methods, we concentrate on one of the most efficient versions known, the primal–dual central path method, and again we do not present the technical machinery in full. Rigorous mathematical statements are clearly distinguished from informal explanations in such parts.

The only real prerequisite to this book is undergraduate linear algebra. We summarize the required notions and results in an appendix. Some of the examples also use rudimentary graph-theoretic terminology, and at several places we refer to notions and facts from calculus; all of these should be a part of standard undergraduate curricula.

Errors. If you find errors in the book, especially serious ones, we would appreciate it if you would let us know (email: matousek@kam.mff.cuni.cz, gaertner@inf.ethz.ch). We plan to post a list of errors at <http://www.inf.ethz.ch/personal/gaertner/lpbook>.

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