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Jacques Istaš

Mathematical Modeling for the Life Sciences

With 31 Figures

 Springer

Jacques Istas
Département IMSS BSHM
Université Pierre Mendès-France
38000 Grenoble
France
e-mail: jacques.istas@umpf-grenoble.fr

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Notations

- \mathbb{R} : set of real numbers.
- \mathbb{N} : set of nonnegative integers.
- \mathbb{Z} : set of integers.
- $C_n^k = \frac{n!}{k!(n-k)!}$: binomial coefficient.
- $\#A$: cardinal number of A .
- $\mathbf{1}_A$: indicator function of A .
- $L^2(\Omega)$: set of square integrable function on Ω .
- $\frac{\partial f(x, y)}{\partial x}$: partial derivative of function f with respect to x .
- $\text{grad}f \equiv \frac{\partial f}{\partial x}\mathbf{i} + \frac{\partial f}{\partial y}\mathbf{j} + \frac{\partial f}{\partial z}\mathbf{k}$ (gradient of f).
- $\nabla J \equiv \frac{\partial J}{\partial x} + \frac{\partial J}{\partial y} + \frac{\partial J}{\partial z}$ (divergence of J).
- $\Delta f \equiv \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$ (Laplacian of f).
- $\mathbf{E}X$: expectation of the random variable X .
- $\mathbf{P}(\omega)$: probability of the event ω .
- $\text{var}(X)$: variance of the random variable X .
- i.i.d. r.v.: independent and identically distributed random variables.
- g.c.d.: greatest common divisor.