Engineering Damage Mechanics

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Ductile, Creep, Fatigue and Brittle Failures

With 135 Figures

Springer

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Foreword

The analysis of stress and deformation of structures and components is not an end in itself. The aim is to predict the serviceability, reliability, and manufacturability of both existing structures and components and of proposed designs. In order to make such assessments, the mode (or modes) of failure need to be known and accounted for. The damage can take many forms, e.g., cracks, voids, chemical attack. In any case, the result is deterioration of the structure or component. Predicting the implications of that deterioration for mechanical integrity is the goal of damage mechanics.

The concept of damage and the realization of its importance for engineering are not new. What is relatively new (within the last 35 years or so) is the development of the framework of Continuum Damage Mechanics. There have been many contributors to this development, but the contributions of the French school and particularly those of Jean Lemaitre stand out. The field of damage mechanics has advanced to the point where it is an engineering tool, with wide applications in industry. In practice, needs arise at several levels. In a preliminary design stage, the need is often for rapid methods of analysis that exhibit trends. In a final design stage or in carrying out a safety assessment the need can be for accurate quantitative predictions. This book provides formulations that span such a range for a variety of technologically important modes of failure, providing a perspective on the advantages and disadvantages of various approaches – from uncoupled, post-processing analyses to fully coupled damage analyses.

After introductory chapters on Continuum Damage Mechanics and numerical analysis of damage, the remaining chapters focus on a mode of damage – ductile failures; low-cycle fatigue; creep, creep-fatigue and dynamic failures; high-cycle fatigue; and failure of brittle and quasi-brittle materials. Each of these chapters appropriately begins with a section on engineering considerations to set the stage and provides a guide to analysis methods and tools. It is quite remarkable that such a wide range of behaviors are incorporated within a unified presentation. The damage mechanics "apple" has blossomed into a tree with many branches!

Since failure is a complex nonlinear process, the predicted behavior can be sensitive to parameter values. Their appropriate identification is key for reliable engineering predictions, as is understanding the sensitivity of predictions to the particular choice of parameter values. The presentation here pays attention to parameter sensitivity as well as to parameter identification.

This book provides a comprehensive guide to Engineering Damage Mechanics. It should appeal to all engineers and students of engineering concerned with lifetime prediction and with the failure resistant design of structures, components, and processes.

Brown University, USA *Alan Needleman*

Introduction

The single apple has become a tree, an apple tree painted by Annie Lemaitre from which two apples fell on the cover page! A decade after "A Course on Damage Mechanics" the topic has grown up to reach the field of applications. Aircraft engines and, more generally, aeronautics, nuclear power plants, metal forming, civil engineering, and the automotive industry have already developed and benifited from damage-based methods to increase performance and security. The time has come to propose simplified or more advanced methods, structured in a unified framework to designers of any mechanical components such as early design with fast calculation of structural failures by closed-form solutions and final validation of solutions by numerical failures analysis. This was the ambition for this book!

This is the reason for having many basic examples and insisting on practical methods such as the difficult problem of the material parameters identification for which a systematic sensitivity analysis is performed for each type of application. Very accurate calculations are too often made with a very poor accuracy of the material parameters! To help, probability concepts are introduced either for random loadings or scatter due to microdefects in the materials. This is done mainly for fatigue failure phenomena and brittle materials but may apply to other cases.

Damage mechanics applies to all materials, including metals and alloys, polymers, elastomers, composites, and concrete, because even if the mechanisms are different on a microscale, they have more or less the same qualitative behaviors on meso- or macroscales. Nevertheless, due to data availability most quantitative examples are related to metals.

- The first chapter reassembles the main concepts of Continuum Damage Mechanics, that is the theoretical tools to apply to specific cases: damage variable, isotropic and anisotropic description, thermodynamics which yields methods of damage measurements, damage laws, coupling with strain behavior, localization, and mesocrack initiation.
- The second chapter is a set of numerical tools for solving the nonlinear problems related to damage evaluation in structures. Post-processing clas-

sical structure analysis, either by the time integration of a damage law, solving a micromechanical two-scale damage model, or when damage is not localized, by solving fully coupled strain-damage structural problems.

- The five following chapters are organized in the same way: four sections from the simplest methods with closed-form solutions to more advanced numerical analyses. The first sections “Engineering Considerations” give the domain of application of the chapter. The second sections “Fast Calculations of Structural Failures” describe some simplified methods to be used in early design. They are applied in the third sections “Basic Engineering Examples” to damage failures of members having stress concentration zones, pressurized cylinders and beams in bending. The fourth sections “Numerical Failure Analysis” describe, using examples, more accurate methods for numerical calculations with computers.
  - The third chapter is devoted to ductile failures involving large deformations for applications in metal forming processes or effects of large overloadings on structures in service.
  - The fourth chapter deals with low-cycle fatigue involving important coupling between damage and plasticity for applications on structures heavily burdened by cyclic loadings.
  - The fifth chapter introduces the effects of temperature-inducing creep and its nonlinear interaction with damage for applications on structures loaded statically or cyclically at elevated temperature, or dynamically.
  - The sixth chapter concerns high-cycle fatigue which uses a two-scale damage model of an elasto-plastic damaged inclusion in an elastic matrix with “elastic fatigue” applications from complex histories of loading to three-dimensional and random loadings.
  - The seventh chapter is devoted to brittle and quasi-brittle materials: quasi-brittle when an irreversible process induces damage, brittle when the fracture occurs without any measurable precursor. Statistical and probabilistic methods are used to represent the large scatter generally observed in the failure of these materials. Their applications concern structures made of concrete, ceramics or composite materials.

How should you use the book? As you like it of course but be aware that each chapter is more or less self-contained, with many referrals to the two first chapters of basic concepts of damage mechanics and its numerical processing. Furthermore, at the end of each chapter on applications, the section “Hierachic Approach” is more or less a summary of the chapter with indications on the domain of validity of each model or method. To help engineers, researchers, students, beginners or not, each section is categorized by the number of apples:

🍎 means easy to read, easy to apply.
🍎🍎 means a read with attention and an application with care.
🍎🍎🍎 means a more advanced theory needing a numerical analysis.

Of course this classification is subjective but it has been checked by some friends working mostly in industry: J. Besson from ENSMP Centre des matériaux (Chap. 1), E. Lorentz from Electricité de France (Chap. 2), F. Moussy from Renault (Chap. 3), J.P. Sermage from E.D.F. SEPTEN (Chap. 4), B. Dambrine from SNECMA (Chap. 5), A. Galtier from ARCELOR for (Chap. 6), B. Bary from C.E.A. (Chap. 7), A. Benallal from C.N.R.S.-LMT Cachan and M. Elgueta from Chili University (overall book), and A. Needleman from Brown University who wrote the foreword. Our thanks to all of them for their expertise and advice. "Merci" also to our friends from "Laboratoire de Mécanique et Technologie" at Cachan who participated in the birth of many parts of this book and particularly to Catherine Génin.

Bon courage pour une lecture fructueuse

LMT Cachan, France

Jean Lemaitre
Rodrigue Desmorat

Contents

Notations

It was impossible to avoid using the same letter for different meanings!

Operators

x	scalar
x_i	component of a vector $\vec{x}$
x_{ij}	component of a second order tensor $\boldsymbol{x}$
x_{ijkl}	component of a fourth order tensor $\underline{\boldsymbol{x}}$
$[M]$	matrix
$\dot{x}$	time derivative of x ($\dot{x} = \mathrm{d}x/\mathrm{d}t$)
$x_{i,j}$	gradient of $\vec{x}$ also ∇x
$x_{ij,j}$	divergence of $\boldsymbol{x}$
x_{kk}	trace of $\boldsymbol{x}$
x_H	$\frac{1}{3}$ trace of $\boldsymbol{x}$
x^D_{ij}	component of the deviatoric tensor $\boldsymbol{x}^\mathrm{D}$, $x^\mathrm{D}_{ij} = x_{ij} - x_\mathrm{H}\delta_{ij}$
δ_{ij}	Kronecker delta, $\delta_{ij} = 1$ if $i = j$ and $\delta_{ij} = 0$ if $i \neq j$
$\lvert x \rvert$	absolute value of the scalar x
$\lvert \boldsymbol{x} \rvert_{ij}$	absolute value in terms of principal components of the tensor $\boldsymbol{x}$
$[\![x]\!]$	discontinuity of x
$\langle x \rangle$	Macauley bracket, $\langle x \rangle = x$ if $x \geq 0$ and $\langle x \rangle = 0$ if $x < 0$
$\langle \boldsymbol{x} \rangle^+$ or $\langle \boldsymbol{x} \rangle^+_{ij}$	positive part in terms of principal components of tensor $\boldsymbol{x}$
$\langle \boldsymbol{x} \rangle^-$ or $\langle \boldsymbol{x} \rangle^-_{ij}$	negative part in terms of principal components of tensor $\boldsymbol{x}$
$\overline{x}$	mean value of x
$\overline{\overline{x}}$	standard deviation of the random variable x
Δx	range of x (peak to peak amplitude) or time increment $x_{n+1} - x_n$
$\mathrm{d}, \partial, \delta$	differential operators

$\mathcal{H}$	Heaviside function
ln	neperian logarithm
$\sigma_{ij,j}$	derivative $\frac{\partial}{\partial x_j}\sigma_{ij}$
det	determinant operator
∇	gradient operator
∇^2	laplacian operator

Symbols

$\underline{\boldsymbol{a}}$	rate elasticity tensor
A	damage threshold parameter
A	crack area
$\mathcal{A}_k$	material parameter
A_{D}	Kachanov law damage parameter
$\boldsymbol{A}$	Almansi strain tensor
b, b_{y}	isotropic hardening exponents
b_{S}	Sines criterion parameter
C, C_{y}	kinematic hardening parameters
C_ϵ	heat capacity
C_{h}	specific heat
C_{MC}	Manson–Coffin law parameter
C_{P}	Paris law parameter
$\underline{\boldsymbol{C}}$	dilatation tensor
D	scalar damage variable
D_{ij}	component of the second order damage tensor $\boldsymbol{D}$
D_{T}, D_{S}	transverse and shear damage variables
D_{ijkl}	component of the fourth order damage tensor
D_{c}	critical damage parameter
e^{p}_{ij}	component of the effective plastic strain tensor $\boldsymbol{e}^{\mathrm{p}}$
E	Young modulus of elasticity
E_{ijkl}	component of the elasticity tensor $\underline{\boldsymbol{E}}$
$\boldsymbol{E}$	Green–Lagrange strain tensor
f	yield function of plastic criterion
f_{v}	porosity
F	force
F	dissipative potential function
F_{X}	plastic potential of dissipation
F_{D}	damage potential of dissipation
$\boldsymbol{F}$	gradient of deformation transformation

g	plastic strain-stress function
G	shear elasticity modulus
G	strain energy release rate
G_c	material toughness parameter
h, h_{a}	microdefect closure parameters
H	hardness material parameter
H	activation energy parameter
H_N	creep temperature exponent
$H_{ij} = (\mathbf{1} - \boldsymbol{D})_{ij}^{-1/2}$	component of the damage effective operator $\boldsymbol{H} = (\mathbf{1} - \boldsymbol{D})^{-1/2}$
I_{ijkl}	component of the unit fourth order tensor $\underline{\boldsymbol{I}}$
[Jac]	Jacobian matrix
k	heat conductivity parameter
K	elastic compressibility modulus
K	stress intensity factor
K_{c}	cyclic hardening law coefficient
K_{T}	elastic stress concentration coefficient
k_{Neuber}	Neuber stress concentration correction
K_{p}, K_{p}^{0}, $K_{\mathrm{p}}^{\mathrm{y}}$, $K_{\mathrm{p}}^{\mathrm{f}}$	hardening material parameters
K_{N}, K_{N}^{0}	Norton law parameters
K_{∞}	viscous material parameter
$\boldsymbol{L}$	strain rate tensor
$\underline{\boldsymbol{L}}$	fourth order tangent tensorial operator
m	damage threshold exponent
$moon_i$	Mooney parameters
M, M_0, M_{y}, M_{f}	isotropic hardening exponents
M_{c}	cyclic hardening law exponent
M_{ijkl}	component of the effective operator $\underline{\boldsymbol{M}}$
n	viscous material parameter exponent
$\vec{n}$	unit normal vector
N, n	number of cycles
N_{R}	number of cycles to rupture
N, N_0	Norton law viscous exponents
N_{ij}	shape functions

p	accumulated plastic strain
p_{D}	damage threshold accumulated plastic strain
p_{R}	rupture accumulated plastic strain
P	pressure
$\boldsymbol{P}$	ponderation matrix

q_1, q_2	Gurson law parameters
q	state variable of the reversible domain
Q	stress increase of the reversibility domain
$\vec{q}$	thermal flux vector

r	isotropic hardening state variable
r_{h}	heat source
r_{D}	Kachanov law damage exponent
R	isotropic hardening stress variable
R_∞, R_∞^{y}	saturated isotropic hardening parameters
R_ν, $R_{\nu h}$	triaxiality function

s	specific entropy
s	unified damage law exponent
S	energetic damage law parameter
S	surface
S_{D}	damage surface
Saf	safety factor
$S^{\cdot}_{\mathcal{A}_k}$	sensitivity coefficient
$\boldsymbol{S}$	second Piola–Kirchhoff stress tensor

t	time
t_{R}	rupture time
T	temperature
T_{X}	stress triaxiality, $T_{\mathrm{X}} = \sigma_{\mathrm{H}}/\sigma_{\mathrm{eq}}$

u	displacement
$\vec{U}$	displacement vector
$\{U\}$	nodal displacement vector
$\{U^{\mathrm{e}}\}$	elementary nodal displacement vector

v	wave speed
V	volume
V_0	reference volume of Weibull law
V_{eff}	effective volume of Weibull law

w	energy density
w_D	damage threshold stored energy density
w_e	elastic strain energy density
w_s	stored energy density
W_e	elastic strain energy
W_1, W_2	hyperelatic energy densities
X	uniaxial kinematic back stress
X_{ij}	component of the back stress deviatoric tensor $\boldsymbol{X}$
X_∞, X_∞^y	saturated kinematic hardening material parameter
Y	energy density release rate
Y_{ij}	component of the energy density release rate tensor $\boldsymbol{Y}$
$\overline{Y}$	effective elastic energy density
Z	necking parameter in pure tension
α	dilatation parameter coefficient
$\boldsymbol{\alpha}$	kinematic hardening state variable
β	Eshelby coefficient
γ, γ_y	kinematic hardening material parameter
γ_MC	Manson–Coffin law exponent
$\Gamma(a)$	gamma function: $\Gamma(a) = \int_0^\infty t^{a-1} \exp{(-t)} \mathrm{d}t$
$\gamma(a, x)$	incomplete gamma function: $\gamma(a, x) = \int_0^x t^{a-1} \exp{(-t)} \mathrm{d}t$
δ_0	size of the mesocrack initiated
δ_GTN	Gurson law coalescence parameter
$\boldsymbol{\Delta}$	hypoelastic strain rate tensor
ϵ, ϵ_{ij}, $\boldsymbol{\epsilon}$	uniaxial and tensorial total strains
ϵ_e, ϵ_{ij}^e, $\boldsymbol{\epsilon}^\mathrm{e}$	uniaxial and tensorial elastic strains
ϵ_p, ϵ_{ij}^p, $\boldsymbol{\epsilon}^\mathrm{p}$	uniaxial and tensorial plastic strains
ϵ_pD	damage threshold plastic strain in pure tension
ϵ_pR, $\epsilon_\mathrm{pR}^\star$	rupture plastic strains in pure tension
ϵ_pu	plastic strain for ultimate stress in pure tension
ϵ_R	rupture strain in pure tension
$\epsilon_{\mathrm{p}\Sigma}$	signed equivalent plastic strain
$\boldsymbol{\epsilon}^\pi$	irreversible strain
η	hydrostatic sensitivity damage parameter
η_P	Paris law exponent

ϕ_{D}	damage dissipated energy density
ϕ_{p}	plastic dissipated energy density
ϕ_{Dp}, ϕ_{F}	fracture dissipated energy density
λ	Lamé elastic parameter
λ	elongation
$\dot{\lambda}$	plastic multiplier
μ	Lamé elastic parameter in shear
$\dot{\mu}$	internal sliding multiplier
ν	Poisson ratio of elastic contraction
$\vec{\nu}$	unit reference vector
ν_{ij}	anisotropic contraction ratio
π	cumulative internal sliding
π_{D}	damage threshold
θ	numerical parameter of the θ-method
ρ	mass density
ρ	radius of curvature
σ, σ_{ij}, $\boldsymbol{\sigma}$	uniaxial and tensorial Cauchy stresses
$\tilde{\sigma}$, $\tilde{\sigma}_{ij}$, $\tilde{\boldsymbol{\sigma}}$	uniaxial and tensorial effective stresses
$\sigma^{\mathrm{\mu}}$, $\sigma_{ij}^{\mathrm{\mu}}$, $\boldsymbol{\sigma}^{\mathrm{\mu}}$	stresses at microscale
σ_{H}	hydrostatic stress, $\sigma_{\mathrm{H}} = \frac{1}{3}\sigma_{\mathrm{kk}}$
σ_{eq}	von Mises equivalent stress
σ_{Σ}	signed von Mises stress
$\sigma^{\star}$	damage equivalent stress, $\sigma^{\star} = \sigma_{\mathrm{eq}} R_{\nu}^{1/2}$
σ_{n}	nominal stress
σ_{v}	viscous stress
σ_{R}	rupture stress
σ_{u}	ultimate stress
σ_{y}	yield stress
σ_{y02}	engineering yield stress for $\epsilon_{\mathrm{p}} = 0.2 \cdot 10^{-2}$
σ_{f}	engineering fatigue limit at 10^6 or 10^7 cycles
$\sigma_{\mathrm{f}}^{\infty}$	asymptotic fatigue limit
σ_{s}	reversibility threshold
Σ_{ij}	component of normalized stress tensor $\boldsymbol{\Sigma}$
ψ	Helmholtz specific free energy

ψ_{e}	elastic state potential
ψ_{p}	plastic state potential
ψ_{T}	thermal state potential
$\psi^{\star}$	Gibbs specific free enthalpy
$\psi_{\mathrm{e}}^{\star}$	elastic specific free enthalpy