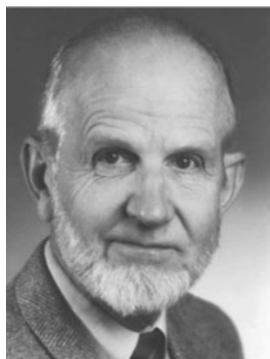


Classics in Mathematics

Lars Hörmander

The Analysis of Linear Partial
Differential Operators II



Born on January 24, 1931, on the southern coast of Sweden, Lars Hörmander did his secondary schooling as well as his undergraduate and doctoral studies in Lund. His principle teacher and adviser at the University of Lund was Marcel Riesz until he returned, then Lars Gårding. In 1956 he worked in the USA, at the universities of Chicago, Kansas, Minnesota and New York, before returning to a chair at the University of Stockholm. He remained a frequent visitor to the US, particularly to Stanford and was Professor at the IAS, Princeton from 1964 to 1968. In 1968 he accepted a chair at the University of Lund, Sweden, where, today he is Emeritus Professor.

Hörmander's lifetime work has been devoted to the study of partial differential equations and its applications in complex analysis. In 1962 he was awarded the Fields Medal for his contributions to the general theory of linear partial differential operators. His book *Linear Partial Differential Operators*, published 1963 by Springer in the Grundlehren series, was the first major account of this theory. His four volume text *The Analysis of Linear Partial Differential Operators*, published in the same series 20 years later, illustrates the vast expansion of the subject in that period.

Lars Hörmander

The Analysis of Linear Partial Differential Operators II

Differential Operators
with Constant Coefficients

Reprint of the 1983 Edition



Springer

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Originally published as Vol. 257 in the series:
Grundlehren der mathematischen Wissenschaften

Library of Congress Control Number: 2004097173

Mathematics Subject Classification (2000):
35B, 35C, 35E, 35G, 35L, 35H10, 35P25, 44A35

ISSN 1431-0821

ISBN 3-540-22516-1 Springer Berlin Heidelberg New York

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Printed in Germany

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Printed on acid-free paper

41/3142YL-54 3 2 1 0

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Springer-Verlag
Berlin Heidelberg New York
London Paris Tokyo Hong Kong

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Box 118
S-22100 Lund
Sweden

With 7 Figures

Second revised printing 1990

AMS Subject Classification: 35 E; 35 G, 35 H, 35 L, 35 P, 44 A 35

ISBN 3-540-12139-0 Springer-Verlag Berlin Heidelberg New York Tokyo
ISBN 0-387-12139-0 Springer-Verlag New York Heidelberg Berlin Tokyo

Library of Congress Cataloging-in-Publication Data

Hörmander, Lars. The analysis of linear partial differential operators / Lars Hörmander. – Rev. ed. p. cm. – (Grundlehren der mathematischen Wissenschaften ; 257) Includes bibliographical references. Contents: 2. Differential operators with constant coefficients.

ISBN (invalid) 0-387-12139-0 (U.S. : v. 2)

1. Differential equations, Partial. 2. Partial differential operators. I. Title. II. Series.

QA377.H578 1990 515'.353–dc20 89-26134 CIP

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© Springer-Verlag Berlin Heidelberg 1983

Printed in Germany

Typesetting: Universitätsdruckerei H. Stürtz AG, 8700 Würzburg

Printing: Reprotechnik Deutschland GmbH, Heidelberg

Bookbinding: Schäffer, Grünstadt

2141/3111 - 543 Printed on acid-free paper

Preface

This volume is an expanded version of Chapters III, IV, V and VII of my 1963 book "Linear partial differential operators". In addition there is an entirely new chapter on convolution equations, one on scattering theory, and one on methods from the theory of analytic functions of several complex variables. The latter is somewhat limited in scope though since it seems superfluous to duplicate the monographs by Ehrenpreis and by Palamodov on this subject.

The reader is assumed to be familiar with distribution theory as presented in Volume I. Most topics discussed here have in fact been encountered in Volume I in special cases, which should provide the necessary motivation and background for a more systematic and precise exposition. The main technical tool in this volume is the Fourier-Laplace transformation. More powerful methods for the study of operators with variable coefficients will be developed in Volume III. However, the constant coefficient theory has given the guidelines for all that work. Although the field is no longer very active – perhaps because of its advanced state of development – and although it is possible to pass directly from Volume I to Volume III, the material presented here should not be neglected by the serious student who wants to gain a balanced perspective of the theory of linear partial differential equations.

I would like to thank all who have helped me in various ways during the preparation of this volume. As in the case of the first Volume I am particularly indebted to Niels Jørgen Kokholm of the University of Copenhagen who has read all the proofs and in doing so suggested many improvements of the text.

Lund in February 1983

Lars Hörmander

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