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Dynamics of Rods

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With 189 Figures

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Preface

The volume is devoted to the dynamics of rods, which is a branch of mechanics of deformable bodies. The main goal of the book is to present systematically theoretical fundamentals of the mechanics of rods as well as numerical methods used for practical purposes.

Linear and nonlinear equations governing a rod's oscillations are presented. Methods of determining eigenvalues and eigenfunctions in conservative and non-conservative problems along with numerical methods dealing with forced, parametric, and random oscillations of rods are given. Some issues of interaction of rods with air (liquid) flows and the dynamics of space-curved rods containing flows of liquid are considered.

The book consists of nine chapters and appendices and may be conventionally divided into two parts. That is, Chapters 1 to 6 contain, in the main, theoretical material, whereas Chapters 7 to 9 illustrate the application of the theoretical results to problems of practical interest. Problems for self-study are found in Chapters 3, 5, and 7. The solutions to most of the problems are given in Appendix B.

The monograph is addressed to undergraduate and postgraduate students and teaching staff of technical universities. It may also be useful for scientists and mechanical engineers working in a wide range of industries.

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Valery Svetlitsky

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