Scientific Computation

Pierre Sagaut

Large Eddy Simulation for Incompressible Flows

An Introduction

Third Edition
With a Foreword by Massimo Germano

With 99 Figures and 15 Tables

Springer

Prof. Dr. Pierre Sagaut
LMM-UPMC/CNRS
Boite 162, 4 place Jussieu
75252 Paris Cedex 05, France
sagaut@lmm.jussieu.fr

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Foreword to the Third Edition

It is with a sense of great satisfaction that I write these lines introducing the third edition of Pierre Sagaut's account of the field of Large Eddy Simulation for Incompressible Flows. Large Eddy Simulation has evolved into a powerful tool of central importance in the study of turbulence, and this meticulously assembled and significantly enlarged description of the many aspects of LES will be a most welcome addition to the bookshelves of scientists and engineers in fluid mechanics, LES practitioners, and students of turbulence in general.

Hydrodynamic turbulence continues to be a fundamental challenge for scientists striving to understand fluid motions in fields as diverse as oceanography, acoustics, meteorology and astrophysics. The challenge also has socioeconomic attributes as engineers aim at predicting flows to control their features, and to improve thermo-fluid equipment design. Drag reduction in external aerodynamics or convective heat transfer augmentation are well-known examples. The fundamental challenges posed by turbulence to scientists and engineers have not, in essence, changed since the appearance of the second edition of this book, a mere two years ago. What has evolved significantly is the field of Large Eddy Simulation (LES), including methods developed to address the closure problem associated with LES (also called the problem of subgrid-scale modeling), numerical techniques for particular applications, and more explicit accounts of the interplay between numerical techniques and subgrid modeling.

The original hope for LES was that simple closures would be appropriate, such as mixing length models with a single, universally applicable model parameter. Kolmogorov's phenomenological theory of turbulence in fact supports this hope but only if the length-scale associated with the numerical resolution of LES falls well within the ideal inertial range of turbulence, in flows at very high Reynolds numbers. Typical applications of LES most often violate this requirement and the resolution length-scale is often close to some externally imposed scale of physical relevance, leading to loss of universality and the need for more advanced, and often much more complex, closure models. Fortunately, the LES modeler disposes of large amount of raw materials from which to assemble improved models. During LES, the resolved motions present rich multi-scale fields and dynamics including highly non-trivial nonlinear interactions which can be interrogated to learn about the local state of turbulence. This availability of dynamical information has led to the formulation of a continuously growing number of different closure models and methodologies and associated numerical approaches, including many variations on several basic themes. In consequence, the literature on LES has increased significantly in recent years. Just to mention a quantitative measure of this trend, in 2000 the ISI science citation index listed 164 papers published including the keywords "large-eddy-simulation" during that year. By 2004 this number had doubled to over 320 per year. It is clear, then, that a significantly enlarged version of Sagaut's book, encompassing much of what has been added to the literature since the book's second edition, is a most welcome contribution to the field.

What are the main aspects in which this third edition has been enlarged compared to the first two? Sagaut has added significantly new material in a number of areas. To begin, the introductory chapter is enriched with an overview of the structure of the book, including an illuminating description of three fundamental errors one incurs when attempting to solve fluid mechanics' infinite-dimensional, non-linear differential equations, namely projection error, discretization error, and in the case of turbulence and LES, the physically very important resolution error. Following the chapters describing in significant detail the relevant foundational aspects of filtering in LES, Sagaut has added a new section dealing with alternative mathematical formulations of LES. These include statistical approaches that replace spatial filtering with conditionally averaging the unresolved motions, and alternative model equations in which the Navier-Stokes equations are replaced with mathematically better behaved equations such as the Leray model in which the advection velocity is regularized (i.e. filtered).

In the chapter dealing with functional modeling approaches, in which the subgrid-scale stresses are expressed in terms of local functionals of the resolved velocity gradients, a more complete account of the various versions of the dynamic model is given, as well as extended discussions of new structure-function and multiscale models. The chapter on structural modeling, in which the stress tensor is reconstructed based on its definition and various direct hypotheses about the small-scale velocity field is significantly enhanced: Closures in which full prognostic transport equations are solved for the subgrid-scale stress tensor are reviewed in detail, and entire new subsections have been added dealing with filtered density function models, with one-dimensional turbulence mapping models, and variational multi-scale models, among others. The chapter focussing on numerical techniques contains an interesting new description of the effects of pre-filtering and of the various methods to perform grid refinement. In the chapter on analysis and validation of LES, a new detailed account is given about methods to evaluate the subgrid-scale kinetic energy. The description of boundary and inflow conditions for LES is enhanced with new material dealing with one-dimensional-turbulence models near walls as well as stochastic tools to generate and modulate random fields for inlet turbulence specification. Chapters dealing with coupling of multiresolution, multidomain, and adaptive grid refinement techniques, as well as LES - RANS coupling, have been extended to include recent additions to the literature. Among others, these are areas to which Sagaut and his co-workers have made significant research contributions.

The most notable additions are two entirely new chapters at the end of the book, on the prediction of scalars using LES. Both passive scalars, for which subgrid-scale mixing is an important issue, and active scalars, of great importance to geophysical flows, are treated. The geophysics literature on LES of stably and unstably stratified flows is voluminous - the field of LES in fact traces its origins to simulating atmospheric boundary layer flows in the early 1970s. Sagaut summarizes this vast field using his classifications of subgrid closures introduced earlier, and the result is a conceptually elegant and concise treatment, which will be of significant interest to both engineering and geophysics practitioners of LES.

The connection to geophysical flow prediction reminds us of the importance of LES and subgrid modeling from a broader viewpoint. For the field of large-scale numerical simulation of complex multiscale nonlinear systems is, today, at the center of scientific discussions with important societal and political dimensions. This is most visible in the discussions surrounding the trustworthiness of global change models. Among others, these include boundary-layer parameterizations that can be studied by means of LES done at smaller scales. And LES of turbulence is itself a prime example of large-scale computing applied to prediction of a multi-scale complex system, including issues surrounding the verification of its predictive capabilities, the testing of the cumulative accuracy of individual building blocks, and interesting issues on the interplay of stochastic and deterministic aspects of the problem. Thus the book - as well as its subject - Large Eddy Simulation of Incompressible Flow, has much to offer to one of the most pressing issues of our times.

With this latest edition, Pierre Sagaut has fully solidified his position as the preeminent cartographer of the complex and multifaceted world of LES. By mapping out the field in meticulous fashion, Sagaut's work can indeed be regarded as a detailed and evolving atlas of the world of LES. And yet, it is not a tourist guide: as with any relatively young terrain in which the main routes have not yet been firmly established, what is called for is unbiased, objective, and sophisticated cartography. The cartographer describes the topography, scenery, and landmarks as they appear, without attempting to preach to the traveler which route is best. In return, the traveler is expected to bring along a certain sophistication to interpret the maps and to discern which among the many paths will most likely lead towards particular destinations of interest. The reader of this latest edition will thus be rewarded with a most solid, insightful, and up-to-date account of an important and exciting field of research.

Baltimore, January 2005 *Charles Meneveau*

Foreword to the Second Edition

It is a particular pleasure to present the second edition of the book on Large Eddy Simulation for Incompressible Flows written by Pierre Sagaut: two editions in two years means that the interest in the topic is strong and that a book on it was indeed required. Compared to the first one, this second edition is a greatly enriched version, motivated both by the increasing theoretical interest in Large Eddy Simulation (LES) and the increasing numbers of applications and practical issues. A typical one is the need to decrease the computational cost, and this has motivated two entirely new chapters devoted to the coupling of LES with multiresolution multidomain techniques and to the new hybrid approaches that relate the LES procedures to the classical statistical methods based on the Reynolds Averaged Navier–Stokes equations.

Not that literature on LES is scarce. There are many article reviews and conference proceedings on it, but the book by Sagaut is the first that organizes a topic that by its peculiar nature is at the crossroads of various interests and techniques: first of all the physics of turbulence and its different levels of description, then the computational aspects, and finally the applications that involve a lot of different technical fields. All that has produced, particularly during the last decade, an enormous number of publications scattered over scientific journals, technical notes, and symposium acta, and to select and classify with a systematic approach all this material is a real challenge. Also, by assuming, as the writer does, that the reader has a basic knowledge of fluid mechanics and applied mathematics, it is clear that to introduce the procedures presently adopted in the large eddy simulation of turbulent flows is a difficult task in itself. First of all, there is no accepted universal definition of what LES really is. It seems that LES covers everything that lies between RANS, the classical statistical picture of turbulence based on the Reynolds Averaged Navier–Stokes equations, and DNS, the Direct Numerical Simulations resolved in all details, but till now there has not been a general unified theory that gradually goes from one description to the other. Moreover we should note the different importance that the practitioners of LES attribute to the numerical and the modeling aspects. At one end the supporters of the *no model* way of thinking argue that the numerical scheme should and could capture by itself the resolved scales. At the other end the theoretical modelers try to develop new universal equations for the filtered quantities. In some cases LES is regarded as a technique imposed by the present provisional inability of the computers to solve all the details. Others think that LES modeling is a contribution to the understanding of turbulence and the interactions among different ideas are often poor.

Pierre Sagaut has elaborated on this immense material with an open mind and in an exceptionally clear way. After three chapters devoted to the basic problem of the scale separation and its application to the Navier–Stokes equations, he classifies the various subgrid models presently in use as functional and structural ones. The chapters devoted to this general review are of the utmost interest: obviously some selection has been done, but both the student and the professional engineer will find there a clear unbiased exposition. After this first part devoted to the fundamentals a second part covers many of the interdisciplinary problems created by the practical use of LES and its coupling with the numerical techniques. These subjects, very important obviously from the practical point of view, are also very rich in theoretical aspects, and one great merit of Sagaut is that he presents them always in an attractive way without reducing the exposition to a mere set of instructions. The interpretation of the numerical solutions, the validation and the comparison of LES databases, the general problem of the boundary conditions are mathematically, physically and numerically analyzed in great detail, with a principal interest in the general aspects. Two entirely new chapters are devoted to the coupling of LES with multidomain techniques, a topic in which Pierre Sagaut and his group have made important contributions, and to the new hybrid approaches RANS/LES, and finally in the last expanded chapter, enriched by new examples and beautiful figures, we have a review of the different applications of LES in the nuclear, aeronautical, chemical and automotive fields.

Both for graduate students and for scientists this book is a very important reference. People involved in the large eddy simulation of turbulent flows will find a useful introduction to the topic and a complete and systematic overview of the many different modeling procedures. At present their number is very high and in the last chapter the author tries to draw some conclusions concerning their efficiency, but probably the person who is only interested in the basic question "*What is the best model for LES?*" will remain a little disappointed. As remarked by the author, both the structural and the functional models have their advantages and disadvantages that make them seem complementary, and probably a mixed modeling procedure will be in the future a good compromise. But for a textbook this is not the main point. The fortunes and the misfortunes of a model are not so simple to predict, and its success is in many cases due to many particular reasons. The results are obviously the most important test, but they also have to be considered in a textbook with a certain reserve, in the higher interest of a presentation that tries as much as possible to be not only systematic but also rational.

To write a textbook obliges one in some way or another to make judgements, and to transmit ideas, sometimes hidden in procedures that for some reason or another have not till now received interest from the various groups involved in LES and have not been explored in full detail.

Pierre Sagaut has succeeded exceptionally well in doing that. One reason for the success is that the author is curious about every detail. The final task is obviously to provide a good and systematic introduction to the beginner, as rational as a book devoted to turbulence can be, and to provide useful information for the specialist. The research has, however, its peculiarities, and this book is unambiguously written by a passionate researcher, disposed to explore every problem, to search in all models and in all proposals the germs of new potentially useful ideas. The LES procedures that mix theoretical modeling and numerical computation are often, in an inextricable way, exceptionally rich in complex problems. What about *the problem of the mesh adaptation on unstructured grids for large eddy simulations?* Or *the problem of the comparison of the LES results with reference data? Practice shows that nearly all authors make comparisons with reference data or analyze large eddy simulation data with no processing of the data ....* Pierre Sagaut has the courage to dive deep into procedures that are sometimes very difficult to explore, with the enthusiasm of a genuine researcher interested in all aspects and confident about every contribution. This book now in its second edition seems really destined for a solid and durable success. Not that every aspect of LES is covered: the rapid progress of LES in compressible and reacting flows will shortly, we hope, motivate further additions. Other developments will probably justify new sections. What seems, however, more important is that the basic style of this book is exceptionally valid and open to the future of a young, rapidly evolving discipline. This book is not an encyclopedia and it is not simply a monograph, it provides a framework that can be used as a text of lectures or can be used as a detailed and accurate review of modeling procedures. The references, now increased in number to nearly 500, are given not only to extend but largely to support the material presented, and in some cases the dialogue goes beyond the original paper. As such, the book is recommended as a fundamental work for people interested in LES: the graduate and postgraduate students will find an immense number of stimulating issues, and the specialists, researchers and engineers involved in the more and more numerous fields of application of LES will find a reasoned and systematic handbook of different procedures. Last, but not least, the applied mathematician can finally enjoy considering the richness of challenging and attractive problems proposed as a result of the interaction among different topics.

Torino, April 2002 *Massimo Germano*

Foreword to the First Edition

Still today, turbulence in fluids is considered as one of the most difficult problems of modern physics. Yet we are quite far from the complexity of microscopic molecular physics, since we only deal with Newtonian mechanics laws applied to a continuum, in which the effect of molecular fluctuations has been smoothed out and is represented by molecular-viscosity coefficients. Such a system has a dual behaviour of determinism in the Laplacian sense, and extreme sensitivity to initial conditions because of its very strong non-linear character. One does not know, for instance, how to predict the critical Reynolds number of transition to turbulence in a pipe, nor how to compute precisely the drag of a car or an aircraft, even with today's largest computers.

We know, since the meteorologist Richardson,[1] numerical schemes allowing us to solve in a deterministic manner the equations of motion, starting with a given initial state and with prescribed boundary conditions. They are based on momentum and energy balances. However, such a resolution requires formidable computing power, and is only possible for low Reynolds numbers. These Direct-Numerical Simulations may involve calculating the interaction of several million interacting sites. Generally, industrial, natural, or experimental configurations involve Reynolds numbers that are far too large to allow direct simulations,[2] and the only possibility then is Large Eddy Simulations, where the small-scale turbulent fluctuations are themselves smoothed out and modelled via eddy-viscosity and diffusivity assumptions. The history of large eddy simulations began in the 1960s with the famous Smagorinsky model. Smagorinsky, also a meteorologist, wanted to represent the effects upon large synoptic quasi-two-dimensional atmospheric or oceanic motions[3] of a three-dimensional subgrid turbulence cascading toward small scales according to mechanisms described by Richardson in 1926 and formalized by the famous mathematician Kolmogorov in 1941.[4] It is interesting to note that Smagorinsky's model was a total failure as far as the

[1] L.F. Richardson, *Weather Prediction by Numerical Process*, Cambridge University Press (1922).

[2] More than 10^{15} modes should be necessary for a supersonic-plane wing!

[3] Subject to vigorous inverse-energy cascades.

[4] L.F. Richardson, Proc. Roy. Soc. London, Ser A, **110**, pp. 709–737 (1926); A. Kolmogorov, Dokl. Akad. Nauk SSSR, **30**, pp. 301–305 (1941).

atmosphere and oceans are concerned, because it dissipates the large-scale motions too much. It was an immense success, though, with users interested in industrial-flow applications, which shows that the outcomes of research are as unpredictable as turbulence itself! A little later, in the 1970s, the theoretical physicist Kraichnan[5] developed the important concept of spectral eddy viscosity, which allows us to go beyond the separation-scale assumption inherent in the typical eddy-viscosity concept of Smagorinsky. From then on, the history of large eddy simulations developed, first in the wake of two schools: Stanford–Torino, where a dynamic version of Smagorinsky's model was developed; and Grenoble, which followed Kraichnan's footsteps. Then researchers, including industrial researchers, all around the world became infatuated with these techniques, being aware of the limits of classical modeling methods based on the averaged equations of motion (Reynolds equations).

It is a complete account of this young but very rich discipline, the large eddy simulation of turbulence, which is proposed to us by the young ONERA researcher Pierre Sagaut, in a book whose reading brings pleasure and interest. *Large-Eddy Simulation for Incompressible Flows - An Introduction* very wisely limits itself to the case of incompressible fluids, which is a suitable starting point if one wants to avoid multiplying difficulties. Let us point out, however, that compressible flows quite often exhibit near-incompressible properties in boundary layers, once the variation of the molecular viscosity with the temperature has been taken into account, as predicted by Morkovin in his famous hypothesis.[6] Pierre Sagaut shows an impressive culture, describing exhaustively all the subgrid-modeling methods for simulating the large scales of turbulence, without hesitating to give the mathematical details needed for a proper understanding of the subject.

After a general introduction, he presents and discusses the various filters used, in cases of statistically homogeneous and inhomogeneous turbulence, and their applications to Navier–Stokes equations. He very aptly describes the representation of various tensors in Fourier space, Germano-type relations obtained by double filtering, and the consequences of Galilean invariance of the equations. He then goes into the various ways of modeling isotropic turbulence. This is done first in Fourier space, with the essential wave-vector triad idea, and a discussion of the transfer-localness concept. An excellent review of spectral-viscosity models is provided, with developments going beyond the original papers. Then he goes to physical space, with a discussion of the structure-function models and the dynamic procedures (Eulerian and Lagrangian, with energy equations and so forth). The study is then generalized to the anisotropic case. Finally, functional approaches based on Taylor series expansions are discussed, along with non-linear models, homogenization techniques, and simple and dynamic mixed models.

[5] He worked as a postdoctoral student with Einstein at Princeton.

[6] M.V. Morkovin, in *Mécanique de la Turbulence*, A. Favre et al. (eds.), CNRS, pp. 367–380 (1962).

Pierre Sagaut also discusses the importance of numerical errors, and proposes a very interesting review of the different wall models in the boundary layers. The last chapter gives a few examples of applications carried out at ONERA and a few other French laboratories. These examples are well chosen in order of increasing complexity: isotropic turbulence, with the non-linear condensation of vorticity into the "worms" vortices discovered by Siggia;[7] planar Poiseuille flow with ejection of "hairpin" vortices above low-speed streaks; the round jet and its alternate pairing of vortex rings; and, finally, the backward-facing step, the unavoidable test case of computational fluid dynamics. Also on the menu: beautiful visualizations of separation behind a wing at high incidence, with the shedding of superb longitudinal vortices. Completing the work are two appendices on the statistical and spectral analysis of turbulence, as well as isotropic and anisotropic EDQNM modeling.

A bold explorer, Pierre Sagaut had the daring to plunge into the jungle of multiple modern techniques of large-scale simulation of turbulence. He came back from his trek with an extremely complete synthesis of all the models, giving us a very complete handbook that novices can use to start off on this enthralling adventure, while specialists can discover models different from those they use every day. *Large-Eddy Simulation for Incompressible Flows - An Introduction* is a thrilling work in a somewhat austere wrapping. I very warmly recommend it to the broad public of postgraduate students, researchers, and engineers interested in fluid mechanics and its applications in numerous fields such as aerodynamics, combustion, energetics, and the environment.

Grenoble, March 2000 *Marcel Lesieur*

[7] E.D. Siggia, J. Fluid Mech., **107**, pp. 375–406 (1981).

Preface to the Third Edition

Working on the manuscript of the third edition of this book was a very exciting task, since a lot of new developments have been published since the second edition was printed.

The large-eddy simulation (LES) technique is now recognized as a powerful tool and real applications in several engineering fields are more and more frequently found. This increasing demand for efficient LES tools also sustains growing theoretical research on many aspects of LES, some of which are included in this book. Among them, it is worth noting the mathematical models of LES (the convolution filter being only one possiblity), the definition of boundary conditions, the coupling with numerical errors, and, of course, the problem of defining adequate subgrid models. All these issues are discussed in more detail in this new edition. Some good news is that other monographs, which are good complements to the present book, are now available, showing that LES is a topic with a fastly growing audience. The reader interested in mathematics-oriented discussions will find many details in the monoghaphs by Volker John (*Large-Eddy Simulation of Turbulent Incompressible Flows*, Springer) and Berselli, Illiescu and Layton (*Mathematics of Large-Eddy Simulation of Turbulent Flows*, Springer), while people looking for a subsequent description of numerical methods for LES and direct numerical simulation will enjoy the book by Bernard Geurts (*Elements of Direct and Large-Eddy Simulation*, Edwards). More monographs devoted to particular features of LES (implicit LES appraoches, mathematical backgrounds, etc.) are to come in the near future.

My purpose while writing this third edition was still to provide the reader with an up-to-date review of existing methods, approaches and models for LES of incompressible flows. All chapters of the previous edition have been updated, with the hope that this nearly exhaustive review will help interested readers avoid rediscovering old things. I would like to apologize in advance for certainly forgetting some developments. Two entirely new chapters have been added. The first one deals with mathematical models for LES. Here, I believe that the interesting point is that the filtering approach is nothing but a model for the true LES problem, and other models have been developed that seem to be at least as promising as this very popular one. The second new chapter is dedicated to the scalar equation, with both passive scalar and active scalar (stable/unstable stratification effects) cases being discussed. This extension illustrates the way the usual LES can be extended and how new physical mechanisms can be dealt with, but also inspires new problems.

Paris, November 2004 *Pierre Sagaut*

Preface to the Second Edition

The astonishingly rapid development of the Large-Eddy Simulation technique during the last two or three years, both from the theoretical and applied points of view, have rendered the first edition of this book lacunary in some ways. Three to four years ago, when I was working on the manuscript of the first edition, coupling between LES and multiresolution/multilevel techniques was just an emerging idea. Nowadays, several applications of this approach have been succesfully developed and applied to several flow configurations. Another example of interest from this exponentially growing field is the development of hybrid RANS/LES approaches, which have been derived under many different forms. Because these topics are promising and seem to be possible ways of enhancing the applicability of LES, I felt that they should be incorporated in a general presentation of LES.

Recent developments in LES theory also deal with older topics which have been intensely revisited by reseachers: a unified theory for deconvolution and scale similarity ways of modeling have now been established; the "no model" approach, popularized as the MILES approach, is now based on a deeper theoretical analysis; a lot of attention has been paid to the problem of the definition of boundary conditions for LES; filtering has been extended to Navier–Stokes equations in general coordinates and to Eulerian time–domain filtering.

Another important fact is that LES is now used as an engineering tool for several types of applications, mainly dealing with massively separated flows in complex configurations. The growing need for unsteady, accurate simulations, more and more associated with multidisciplinary applications such as aeroacoustics, is a very powerful driver for LES, and it is certain that this technique is of great promise.

For all these reasons, I accepted the opportunity to revise and to augment this book when Springer offered it me. I would also like to emphasize the fruitful interactions between "traditional" LES researchers and mathematicians that have very recently been developed, yielding, for example, a better understanding of the problem of boundary conditions. Mathematical foundations for LES are under development, and will not be presented in this book, because I did not want to include specialized functional analysis discussions in the present framework.

I am indebted to an increasing number of people, but I would like to express special thanks to all my colleagues at ONERA who worked with me on LES: Drs. E. Garnier, E. Labourasse, I. Mary, P. Quéméré and M. Terracol. All the people who provided me with material dealing with their research are also warmly acknowledged. I also would like to thank all the readers of the first edition of this book who very kindly provided me with their remarks, comments and suggestions. Mrs. J. Ryan is once again gratefully acknowledged for her help in writing the English version.

Paris, April 2002 *Pierre Sagaut*

Preface to the First Edition

While giving lectures dealing with Large-Eddy Simulation (LES) to students or senior scientists, I have found difficulties indicating published references which can serve as general and complete introductions to this technique.

I have tried therefore to write a textbook which can be used by students or researchers showing theoretical and practical aspects of the Large Eddy Simulation technique, with the purpose of presenting the main theoretical problems and ways of modeling. It assumes that the reader possesses a basic knowledge of fluid mechanics and applied mathematics.

Introducing Large Eddy Simulation is not an easy task, since no unified and universally accepted theoretical framework exists for it. It should be remembered that the first LES computations were carried out in the early 1960s, but the first rigorous derivation of the LES governing equations in general coordinates was published in 1995! Many reasons can be invoked to explain this lack of a unified framework. Among them, the fact that LES stands at the crossroads of physical modeling and numerical analysis is a major point, and only a few really successful interactions between physicists, mathematicians and practitioners have been registered over the past thirty years, each community sticking to its own language and center of interest. Each of these three communities, though producing very interesting work, has not yet provided a complete theoretical framework for LES by its own means. I have tried to gather these different contributions in this book, in an understandable form for readers having a basic background in applied mathematics.

Another difficulty is the very large number of existing physical models, referred to as subgrid models. Most of them are only used by their creators, and appear in a very small number of publications. I made the choice to present a very large number of models, in order to give the reader a good overview of the ways explored. The distinction between functional and structural models is made in this book, in order to provide a general classification; this was necessary to produce an integrated presentation.

In order to provide a useful synthesis of forty years of LES development, I had to make several choices. Firstly, the subject is restricted to incompressible flows, as the theoretical background for compressible flow is less evolved. Secondly, it was necessary to make a unified presentation of a large number of works issued from many research groups, and very often I have had to change the original proof and to reduce it. I hope that the authors will not feel betrayed by the present work. Thirdly, several thousand journal articles and communications dealing with LES can be found, and I had to make a selection. I have deliberately chosen to present a large number of theoretical approaches and physical models to give the reader the most general view of what has been done in each field. I think that the most important contributions are presented in this book, but I am sure that many new physical models and results dealing with theoretical aspects will appear in the near future.

A typical question of people who are discovering LES is "what is the best model for LES?". I have to say that I am convinced that this question cannot be answered nowadays, because no extensive comparisons have been carried out, and I am not even sure that the answer exists, because people do not agree on the criterion to use to define the "best" model. As a consequence, I did not try to rank the model, but gave very generally agreed conclusions on the model efficiency.

A very important point when dealing with LES is the numerical algorithm used to solve the governing equations. It has always been recognized that numerical errors could affect the quality of the solution, but new emphasis has been put on this subject during the last decade, and it seems that things are just beginning. This point appeared as a real problem to me when writing this book, because many conclusions are still controversial (e.g. the possibility of using a second-order accurate numerical scheme or an artificial diffusion). So I chose to mention the problems and the different existing points of view, but avoided writing a part dealing entirely with numerical discretization and time integration, discretization errors, etc. This would have required writing a companion book on numerical methods, and that was beyond the scope of the present work. Many good textbooks on that subject already exist, and the reader should refer to them.

Another point is that the analysis of the coupling of LES with typical numerical techniques, which should greatly increase the range of applications, such as Arbitrary Lagrangian–Eulerian methods, Adaptive Mesh-Refinement or embedded grid techniques, is still to be developed.

I am indebted to a large number of people, but I would like to express special thanks to Dr. P. Le Quére, O. Daube, who gave me the opportunity to write my first manuscript on LES, and to Prof. J.M. Ghidaglia who offered me the possibility of publishing the first version of this book (in French). I would also like to thank ONERA for helping me to write this new, augmented and translated version of the book. Mrs. J. Ryan is gratefully acknowledged for her help in writing the English version.

Paris, September 2000 *Pierre Sagaut*

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