

Scheduling Algorithms

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Scheduling Algorithms

Fourth Edition
with 77 Figures
and 32 Tables



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Preface of the Fourth Edition

In this edition new results have been added to the complexity columns. Furthermore, the bibliography has been updated.

Again many thanks go to Marianne Gausmann for the typesetting and to Dr. Sigrid Knust for taking care of the complexity columns which can be found under the www-address

<http://www.mathematik.uni-osnabrueck.de/research/OR/class>.

Osnabrück, November 2003

Peter Brucker

Preface of the Third Edition

In this edition again the complexity columns at the end of each chapter and the corresponding references have been updated. I would like to express my gratitude to Dr. Sigrid Knust for taking care of a corresponding documentation of complexity results for scheduling problems in the Internet. These pages can be found under the world-wide-web address <http://www.mathematik.uni-osnabrueck.de/research/OR/class>.

In addition to the material of the second edition some new results on scheduling problems with release times and constant processing times and on multiprocessor task problems in which each task needs a certain number of processors have been included.

The new edition has been rewritten in L^AT_EX 2_ε. Many thanks go to Marianne Gausmann for the new typesetting and to Christian Strotmann for creating the bibliography database files.

Osnabrück, March 2001

Peter Brucker

Preface of the Second Edition

In this revised edition new material has been added. In particular, the chapters on batching problems and multiprocessor task scheduling have been augmented. Also the complexity columns at the end of each chapter have been updated. In this connection I would like to thank Jan Karel Lenstra for providing the current results of the program MSPCLASS. I am grateful for the constructive comments of Jacek Blazewicz, Johann Hurink, Sigrid Knust, Svetlana Kravchenko, Erwin Pesch, Maurice Queyranne, Vadim Timkowsky, Jürgen Zimmermann which helped to improve the first edition.

Finally, again special thanks go to Marianne Gausmann and Teresa Gehrs for the T_EX typesetting and for improving the English.

Osnabrück, November 1997

Peter Brucker

Preface

This is a book about scheduling algorithms. The first such algorithms were formulated in the mid fifties. Since then there has been a growing interest in scheduling. During the seventies, computer scientists discovered scheduling as a tool for improving the performance of computer systems. Furthermore, scheduling problems have been investigated and classified with respect to their computational complexity. During the last few years, new and interesting scheduling problems have been formulated in connection with flexible manufacturing.

Most parts of the book are devoted to the discussion of polynomial algorithms. In addition, enumerative procedures based on branch & bound concepts and dynamic programming, as well as local search algorithms, are presented.

The book can be viewed as consisting of three parts. The first part, Chapters 1 through 3, covers basics like an introduction to and classification of scheduling problems, methods of combinatorial optimization that are relevant for the solution procedures, and computational complexity theory.

The second part, Chapters 4 through 6, covers classical scheduling algorithms for solving single machine problems, parallel machine problems, and shop scheduling problems.

The third and final part, Chapters 7 through 11, is devoted to problems discussed in the more recent literature in connection with flexible manufacturing, such as scheduling problems with due dates and batching. Also, multiprocessor task scheduling is discussed.

Since it is not possible to cover the whole area of scheduling in one book, some restrictions are imposed. Firstly, in this book only machine or processor scheduling problems are discussed. Secondly, some interesting topics like cyclic scheduling, scheduling problems with finite input and/or output buffers, and general resource constrained scheduling problems are not covered in this book.

I am indebted to many people who have helped me greatly in preparing this book. Students in my courses during the last three years at the University of Osnabrück have given many suggestions for improving earlier versions of this material. The following people read preliminary drafts of all or part of the book and made constructive comments: Johann Hurink, Sigrid Knust, Andreas Krämer, Wieslaw Kubiak, Helmut Mausser.

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I am grateful to the Deutsche Forschungsgemeinschaft for supporting the research that underlies much of this book. I am also indebted to the Mathematics and Computer Science Department of the University of Osnabrück, the College of Business, University of Colorado at Boulder, and the Computer Science Department, University of California at Riverside for providing me with an excellent environment for writing this book.

Finally, special thanks go to Marianne Gausmann for her tireless efforts in translating my handwritten hieroglyphics and figures into input for the \TeX typesetting system.

Osnabrück, April 1995

Peter Brucker

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