

## Advances in Spatial Science

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Tõnu Puu

# Mathematical Location and Land Use Theory

An Introduction

Second, Revised and Enlarged Edition  
with 114 Figures



Springer

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# Preface to the Second Edition

In his student years the author of this book was fortunate to have Tord Palander, once the leading location theorist of his age, as teacher and later as his thesis adviser. Palander started his economics career in the 30's with taking interest in spatial economics, and ended up with monetary economics in the 50's. The author made the reverse route. So, in the intersection of our professional lives Palander and I both worked with monetary issues, and we never spoke of spatial economics in his remaining lifetime. Despite this the author assimilated Palander's basic philosophy which perspired already through his dissertation "*Beiträge zur Standortstheorie*": Good science is there to be easily understood, the difficult and mysterious being under the constant suspicion of being wrong.

Also, it was when trying to organize Palander's scientific remains upon his death in 1972 that the author found two small papers by Martin Beckmann from the early 50's. These concise contributions, largely overlooked, which did cast the entire spatial trade equilibrium in terms of two partial differential equations became the focus of interest of the author for many years.

It was, however, at first hard to get ahead with the Beckmann model, just because it was so general. It could subsume most of the models ever developed in spatial economics, but, due to this very generality, it was difficult to get out any particular information, except in terms of examples.

It was therefore fortunate when the author also got across some not very widespread results from the generic theory of differential equations, which were largely regarded as dead ends because they did not generalize to dimensions higher than two, and were therefore not useful for physicists. For two dimensional flows in the space economy - of trade, commuters, migrants, or just propagating impacts, such as growth and business cycles - these results provided an invaluable tool for characterizing the stable structures. The same was true about catastrophe theory which provided the means to analyse transitions between the stable structures.

In the 80's the author managed to convince Martin Beckmann to take renewed interest in his model, and we produced a few works together. The battle cry was "to put space back into spatial economics".

The present book draws heavily on these studies, but this time the author considered it important to put all this in the broader context of the classical models of spatial economics. They have an aesthetic and intuitive appeal, and have been sadly ignored by economics of the mainstream. In recent years a few mainstream economists, such as Paul Krugman, have again recognised the importance of taking space in account, but the proper tools for the analysis of these phenomena are still lacking in the curriculum of most economists.

Spatial phenomena are basically visual, and therefore the author also took great pains to illustrate the book by profiting from the means that a modern computer provides.

The second edition is enlarged with several topics which were serious omissions in the first edition. The added material mainly belongs to two topics: 1) Nested markets, central place hierarchies, and rank-size relations, and: 2) Spatial oligopoly in the tradition of Hotelling's seminal 1929 contribution. This issue is dealt with using elastic demand, which removes most of the paradoxes encountered in Hotelling's original study. It is also extended to two-dimensional space, which until now never seems to have been touched upon.

Though the study is theoretical, and therefore claims moderate resources, the isolated location at a poor Swedish university in the North, almost at the Polar Circle, would have made the work impossible, had not The Swedish Social Science Research Council been generous over a very long period, providing funds for travel, books, journals, software, hardware, i.e. all those things that make even theoretical scientific work at all possible.

The work for this second edition has been supported by The Swedish Transport and Communication Research Board (VINNOVA), and has taken place within the precincts of the Urban Design research programme at the Centre for Regional Science of Umeå University (CERUM) at which the author continued his research work after retiring from his chair. The author is much indebted to both. The author also wishes to thank Professor Martin J. Beckmann for valuable comments, and the first student group to which the author taught the course, especially Mr. Anders Stenberg, who detected several remaining errors in the first edition.

Umeå March 2003.  
Tönu Puu

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