Advances in Numerical Mathematics

Gerhard Zumbusch

Parallel Multilevel Methods

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Adaptive Mesh Refinement and Loadbalancing

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Preface

Numerical simulation promises new insight in science and engineering. In addition to the traditional ways to perform research in science, that is laboratory experiments and theoretical work, a third way is being established: *numerical simulation*. It is based on both mathematical models and experiments conducted on a computer. The discipline of scientific computing combines all aspects of numerical simulation. The typical approach in scientific computing includes modelling, numerics and simulation, see Figure 1.

Quite a lot of phenomena in science and engineering can be modelled by partial differential equations (PDEs). In order to produce accurate results, complex models and high resolution simulations are needed. While it is easy to increase the precision of a simulation, the computational cost of doing so is often prohibitive. Highly efficient simulation methods are needed to overcome this problem. This includes three building blocks for computational efficiency, discretisation, solver and computer.

Adaptive mesh refinement, high order and sparse grid methods lead to discretisations of partial differential equations with a low number of degrees of freedom. Multilevel iterative solvers decrease the amount of work per degree of freedom for the solution of discretised equation systems. Massively parallel computers increase the computational power available for a single simulation. However, parallel computers require parallel algorithms and special methods to code them including data distribution and communication, which poses a severe problem for adaptive mesh refinement. Furthermore multilevel solvers have to be specifically tailored so that they can be applied to the adaptive discretisation. Even the efficient implementation of multilevel methods for sequential and parallel computers poses a severe problem. These aspects will be covered in detail in the following chapters.

Last but not least, let me thank all who supported the present work in one way or another. To name but a few, let me begin with my supervisor Prof. M. Griebel, who supported my research over many years and created a research environment which was probably unique at a mathematics department.

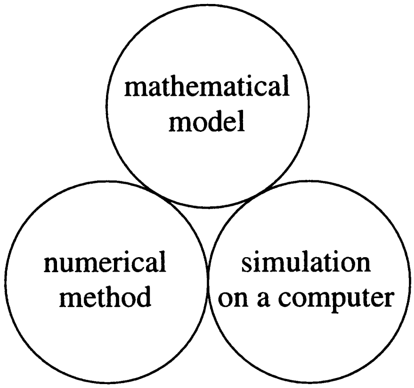

Figure 1. *Three ingredients of scientific computing: a mathematical model, a numerical method and the simulation on a computer.*

This enabled the combination of ideas from fields as diverse as approximation theory and molecular dynamics, multilevel methods and high speed networking. Furthermore, the leading edge equipment allowed for many projects years before it became close being mainstream. However, he also contributed the basic idea of the present work, namely the idea of applying space-filling curve techniques from astrophysical particle methods to parallel adaptive multigrid methods, a topic I worked on ten years ago at TU München then with his support and supervised by Prof. R. Hoppe.

Of course I have to thank the whole group *Scientific Computing and Numerical Simulation*, members of the *Institute for Applied Mathematics* and members of the SFB 256 (Sonderforschungsbereich) *Non-linear Partial Differential Equations.* Let me name some of them individually, e.g. M. A. Schweitzer for the collaboration on the construction of our cluster computing resources and discussions on multigrid methods and parallelisation in general. The sparse grid and wavelet parts were influenced by F. Koster and T. Schiekofer, who calculated some wavelet coefficients for the best approximation results and laid the algorithmic foundations of the finite difference sparse grid discretisation respectively. Some research related to space-filling curves was done by M. Ellerbrake and G. Spahn, who created the pictures of the tetrahedron meshes. The calculations on the T3E at Cray Inc. were supervised by M. Arndt. Furthermore I want to thank Prof. P. Oswald (Lucent) and Prof. H.-J. Bungartz (Stuttgart) for useful discussions on sparse grids and space-filling curves respectively. P. Anderson, M. Arndt, M. Bader, F. Kiefer and M. A. Schweitzer did some proof reading. Thanks also to the referees of the original thesis text and the editors of the book series for their effort of reviewing, their patience, and their comments. I also wish to express my gratitude to Teubner-Verlag for their friendly cooperation.

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Jena, August 2003 *Gerhard Zumbusch*

Contents