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# Combinatorial Optimization Problems in Planning and Decision Making

Theory and Applications

 Springer

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Kyiv, Ukraine  
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Michael Z. Zgurovsky  
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# Abbreviations

AA	Approximation algorithm
AHP	Analytic hierarchy process method by Thomas L. Saaty
APS	Advanced planning and scheduling
CAC	Computer-aided control
CAM	Computer-aided management
CRP	Capacity resources planning
CSRP	Customer synchronized resource planning
DMU	Decision making unit
DRP	Distribution resources planning
E/T	Earliness/tardiness, sum of total earliness and total tardiness
EBSR	Extraction and backward-shifted reinsertion
EFSR	Extraction and forward-shifted reinsertion
ERP	Enterprise resource planning
ERP II	Enterprise resource and relationship processing
FLM	Four-level model of planning (including operative planning) and decision making
JIT	Just in time
LPP	Linear programming problem
MES	Manufacturing execution system
MPS	Master production schedule
MPSS	Maximum priority subsequence
MRP	Material requirements planning
MRP II	Manufacturing resource planning
MSNSP	Multistage network scheduling problem
MSTAA	Machines start times adjustment algorithm
OPF	Orthogonal polynomial of G.E. Forsythe
PCM	Pairwise comparison matrix

PSC-algorithm	An algorithm which includes: sufficient conditions of a feasible solution optimality for which their checking can be implemented only at the stage of a feasible solution construction, and this construction is carried out by a polynomial algorithm (the first polynomial component of the PSC-algorithm); an approximation algorithm with polynomial complexity (the second polynomial component of the PSC-algorithm); also, for NP-hard combinatorial optimization problems, an exact sub-algorithm if sufficient conditions were found, fulfillment of which during the algorithm execution turns it into a polynomial complexity algorithm
SCM	Supply chain management
SSO(s)	Sufficient sign(s) of optimality of a feasible solution
SSPS	Sufficient sign of polynomial solvability of the current iteration
ST	Scheduling theory
TT	Single machine total tardiness minimization problem
TTP	Parallel machines total tardiness minimization problem with a common due date and common fixed start time of machines
TTPL	Parallel machines total tardiness minimization problem with a common due date and arbitrary fixed start times of machines which are less than the due date
TWCT	Single machine total weighted completion time minimization problem with precedence relations given by an oriented acyclic graph
TWCTZ	The TWCT problem for the case when nonzero weights are specified only for the terminal vertices of the oriented graph
TWT	Single machine total weighted tardiness minimization problem
UML	Unified modeling language