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Pantelis Pnigouras

Saturation of the *f*-mode Instability in Neutron Stars

Doctoral Thesis accepted by
the Eberhard-Karls University of Tübingen, Tübingen,
Germany

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Author
Dr. Pantelis Pnigouras
Mathematical Sciences and STAG Research Centre
University of Southampton
Southampton, UK

Supervisor
Prof. Kostas D. Kokkotas
Eberhard-Karls University of Tübingen
Tübingen, Germany

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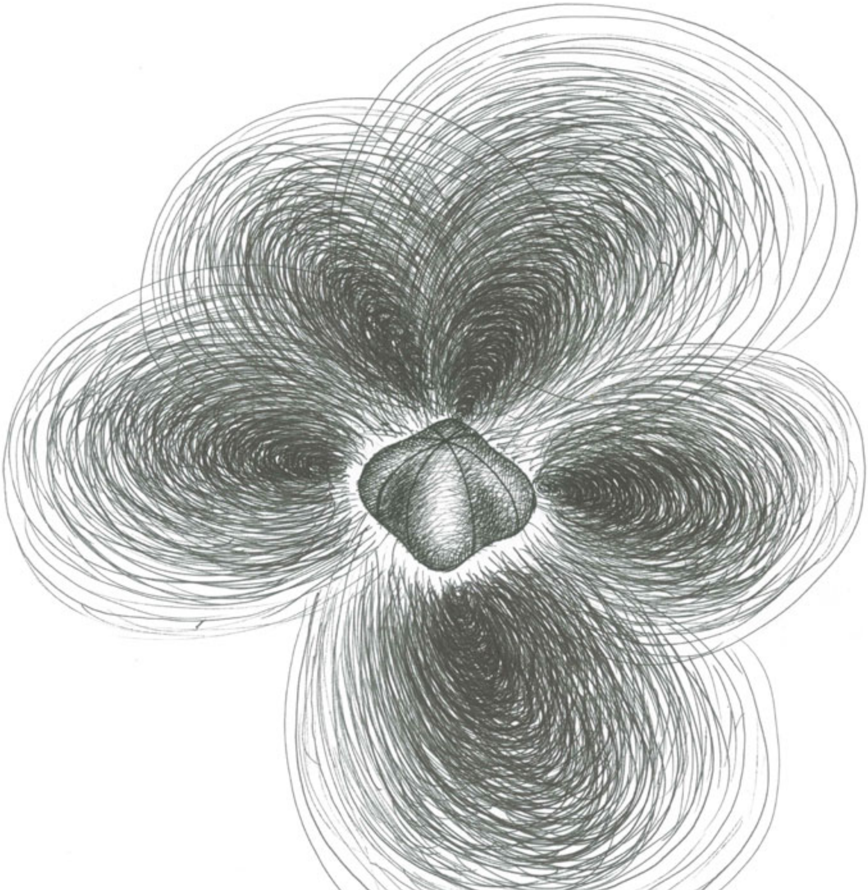

To my father,
for seeking Ithaca.

To my mother,
for relishing the voyage.

Supervisor’s Foreword

The detection of gravitational waves in 2015 signalled a new era for theoretical physics and astrophysics. Thanks to the advanced technology developed, the dark side of the Universe is becoming more transparent to humans, with black holes and neutron stars revealing their violent faces in both the gravitational and the electromagnetic spectrum. It will also not be long until the mystery of the Big Bang will be unveiled via gravitational waves. Last year, for the first time in the history of astronomy, a violent event—the merging of two neutron stars—was observed in all bands of the electromagnetic spectrum, following its discovery via gravitational-wave emission.

There are many fundamental questions which can be tackled with neutron star observations. Two of the most compelling ones relate first of all to the dynamics of neutron stars and second to their efficiency as gravitational-wave sources. By answering these two questions, we expect to reveal the equation of state of matter at supranuclear densities for the first time.

Typical, fast-rotating neutron stars, with masses 1–2 times larger than the solar mass, are expected to be the outcome of the gravitational collapse of a massive star. Furthermore, the merging of two such neutron stars will generically produce, most of the times, a supermassive neutron star. The newly formed object (born from either of the above two formation channels) will be hot and will rotate fast until, possibly, the magnetic field brakes its rotation down to the spins that we observe in the known, old neutron stars. During this early stage, rotational instabilities may take place, deforming the nascent object and emitting copious amounts of gravitational waves at specific frequencies, which will carry information about the physical parameters that describe neutron stars.

One of the oscillation modes that may become unstable due to rotation is the so-called f-mode, which is associated with the fundamental oscillation frequency. The unstable patterns of pulsation of the f-mode grow in time and, when a specific amplitude is reached, they saturate by transferring oscillation energy to other modes via nonlinear mode coupling. The saturation amplitude of the f-mode is directly related to the efficiency with which gravitational waves are emitted. Since 2002 there have been systematic studies about the saturation of another potentially unstable mode, the so-called r-mode. But, until now, there had been no such study for the saturation of the f-mode instability. A plausible reason could be the complicated nature of the problem, compared to similar studies for the r-mode. This thesis is the first systematic and successful attempt in addressing this question. It includes the basic theory of nonlinear mode coupling and develops the methodology for the f-mode. Finally, it arrives at concrete estimations by applying the above to the two promising astrophysical sources of gravitational waves, namely typical and supramassive neutron stars, and examines the possibility of observing the cosmological stochastic background of gravitational waves due to f-mode instabilities throughout the Universe.

The most important application of this work, which is extremely well-timed, is related to the post-merger supramassive neutron star born from the collision, which, apart from its large mass, acquires extremely high spin. This combination favours the onset of rotational instabilities and the neutron star can become unstable for a quite wide range of temperatures and spins. In addition, the instability grows on very short timescales, of the order of a few seconds, during which even the strongest magnetic field will not be able to drain significant amounts of angular momentum. This scenario was studied in detail in Doneva et al. (2015), and these initial results were very promising. The existence of this specific "gravitational-wave afterglow" can be correlated with the observed light curves of short γ-ray bursts, which, in many cases, acquire a plateau lasting hundreds to thousands of seconds and suggesting the survival of the post-merger neutron star remnant for minutes to hours before collapsing to a black hole. During this phase, the star is unstable and the emitted gravitational radiation should be detectable up to a few tens of Mpc with current gravitational-wave detectors, Advanced LIGO and Virgo, and up to a few hundreds of Mpc with the planned next-generation detectors, the Einstein Telescope and the Cosmic Explorer.

I believe that the thesis includes an excellent review on oscillations and instabilities of neutron stars that can be pleasantly read by anyone, while in the appendices one will find the detailed analytic calculations and the extensive formulae derived for the problem.

Concluding, I consider Dr. Pnigouras's thesis an excellent piece of scientific work, written in an elegant, inspiring, and easy-to-read way. The results are sound, timely, and came out of a combination of analytical and computational work.

Tübingen, Germany
June 2018

Prof. Kostas D. Kokkotas

Reference

Doneva, D. D., Kokkotas, K. D. & Pnigouras, P. (2015). Gravitational wave afterglow in binary neutron star mergers. *Physical Review D*, *92*, 104040. https://doi.org/10.1103/PhysRevD.92.124004, arXiv:1510.00673.

Preface

Since their theoretical prediction in 1934 and the serendipitous discovery of the first pulsar in 1967, neutron stars remain among the most challenging objects in the Universe. Thanks to the advancement of theory, experiments, and observations, many aspects of their nature have been deciphered, yet their inner structure is still unknown. Gravitational waves emitted by neutron star oscillations can be used to obtain information about their equation of state, that is, the equation of state of dense nuclear matter. As discovered in the 1970s, certain oscillation modes can be secularly unstable to the emission of gravitational radiation, via the so-called Chandrasekhar-Friedman-Schutz (CFS) mechanism, thus rendering gravitational-wave asteroseismology a promising probe of the neutron star interior, especially after the recent birth of gravitational-wave astronomy.

After its initial growth phase, the instability is expected to saturate, due to nonlinear effects. The saturation amplitude of the unstable mode determines the detectability of the generated gravitational-wave signal, but also affects the evolution of the neutron star through the instability window, namely the region where the instability is active. In this work, we study the saturation of CFS-unstable f-modes (fundamental modes), due to low-order nonlinear mode coupling. Using the quadratic-perturbation approximation, we show that the unstable (parent) mode resonantly couples to pairs of stable (daughter) modes, which drain the parent's energy and make it saturate, via a mechanism called parametric resonance instability. The saturation amplitude of the most unstable f-mode multipoles is calculated throughout their instability windows, for typical and supramassive newborn neutron stars, simply modelled as polytropes in a Newtonian context.

Contrary to previous studies, where the saturation amplitude is treated as a constant, we find that it changes significantly throughout the instability window and, hence, during the neutron star evolution. Using the highest values obtained for the saturation amplitude, a signal from an unstable f-mode may even lie above the sensitivity of current, second-generation, gravitational-wave detectors.

In Chap. 1, we present a brief history of the field and the reasons which motivate such an enterprise, starting from the concept of asteroseismology and how it can be applied in neutron stars, so that the equation of state of dense nuclear matter is determined. Then, we discuss neutron stars as gravitational-wave sources, focusing on the presence of unstable oscillation modes and reviewing their significance both for gravitational-wave asteroseismology and neutron star evolution. In the rest of the chapters, we provide detailed information about the concepts introduced here.

In Chap. 2 we are going to derive the linear perturbation formalism, with the help of which the various classes of modes emerge, like polar (e.g., f-modes) and axial (e.g., r-modes). Chapter 3 is devoted to the f-mode CFS instability, where we will see how the instability works. In Chap. 4, we will obtain the nonlinear perturbation formalism, needed to introduce mode coupling, and discuss the mechanism responsible for the saturation of unstable modes, the so-called parametric resonance instability. The application of the mode coupling analysis to CFS-unstable f-modes, in both typical and supramassive neutron stars governed by polytropic equations of state, is presented in Chap. 5. We should note that, throughout this work, we use Newtonian gravity, with gravitational radiation introduced via post-Newtonian analysis (see Chap. 3). Chapter 6 concludes our study with a summary and some final remarks. At the beginning of each chapter, we review their contents in more detail.

Most of the lengthy derivations of formulae used throughout the chapters are addressed in appendices. In Appendix A, we present the Lane-Emden formalism for polytropic stars, together with Chandrasekhar's extension for rotating configurations. Low-order rotational corrections to the eigenfrequencies and eigenfunctions of polar modes are derived in Appendix B. Explicit formulae for the polar mode growth and damping rates, due to gravitational waves and viscosity, are given in Appendix C. In Appendices D and E, we obtain the equations of motion of nonlinear perturbations and an expression for the polar mode coupling coefficient, respectively. Several important results for a parametrically unstable coupled mode network are derived in Appendix F. Finally, the couplings responsible for the saturation of an unstable f-mode, for one of the models used in Chap. 5, is presented in Appendix G.

This doctoral dissertation is the product of my graduate studies in the Theoretical Astrophysics section of the Institute for Astronomy and Astrophysics in the University of Tübingen, Germany, under the supervision of Prof. Kostas D. Kokkotas. The thesis was submitted on the 11th of July 2016 and the defence took place on the 4th of May 2017. This work resulted in four published papers in refereed journals (Pnigouras and Kokkotas 2015; Doneva et al. 2015; Surace et al. 2016; Pnigouras and Kokkotas 2016) and two papers in conference proceedings (Pnigouras et al. 2016, 2017). Excerpts and figures have been reproduced and reprinted with permission from Pnigouras and Kokkotas (2015), Doneva et al. (2015), and Pnigouras and Kokkotas (2016); copyright (2015, 2016) by the American Physical Society (APS).

The dissertation consists of six main chapters and seven appendices. Sections marked with an asterisk (*) are intended as optional, but some results derived there may still be used in the mandatory sections (albeit always referenced). The same applies to appendices. Following the usual typographic convention, boldface symbols represent vectors.

Computations were performed with an original code, written in Fortran 95 and parallelized with OpenMP, executed in the Theoretical Astrophysics computing centre of the University of Tübingen. Individual runs lasted from a few hours to a few days, depending on the model and the workload. The graphs were produced with Mathematica 10.4.

The title page drawing is an artist's conception of a neutron star, deformed by the hexadecapole f-mode and emitting gravitational waves, courtesy of Christos Tsirvoulis. Finally, the musical score of the Epilogue was created with Noteflight.

Southampton, UK
June 2018

Dr. Pantelis Pnigouras

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Acknowledgements

According to Homer (*Odyssey*, Book II), during the absence of Odysseus from Ithaca, Athena, the goddess of wisdom, disguised herself as Odysseus's friend Mentor, in order to counsel his son Telemachus. I cannot think of a more appropriate simile to describe my relationship with Kostas Kokkotas, who, like a true mentor, never imparted his wisdom in an authoritative way, but conveyed it in the form of advice, always giving me the freedom to question it and learn from my own mistakes. I am grateful not only for his guidance as a teacher, but also for his concern about my well-being and his support.

I would also like to thank all the people who have worked in the Theoretical Astrophysics section of the University of Tübingen during the past few years, and especially Daniela Doneva and Marco Surace, for our fertile collaboration, Kai Schwenzer, for sharing his insight and generously offering his help whenever requested, Marlene Herbrik, for enlightening discussions, Andreas Boden, for tech support, Kostas Glampedakis and Andrea Maselli, for distracting me from the never-ending workload, and Heike Fricke, for explaining the German ways and helping me deal with the ferocious monster that is bureaucracy.

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