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Dynamics of the Unicycle

Modelling and Experimental Verification

Springer

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Preface

As a qualified engineer and a sports enthusiast at the same time, the first author (ML) combines passion with science. The idea concerning the subject of this work appeared when the third author (TK) glimpsed ML unicycling along the university corridor.

The unicycle together with the rider forms a very complex system. It combines mechanics, biomechanics and control theory into a structure which impresses with both its simplicity and improbability. Even more amazing is that the majority of unicyclists do not have the faintest idea that riding such an artefact is, according to science, almost impossible. A similar situation is exemplified by bumblebees unaware of the fact that they should not be able to fly at all.

The present work is devoted to the problem of modelling and control of the 3-D dynamical system consisting of a single-wheel vehicle represented by a unicycle and a cyclist (to be precise—a unicyclist) riding it. The equations of motion were derived with the Boltzmann–Hamel equation in the matrix form, which is based on quasi-velocities and is usually scarcely used. The matrix form enables automatic generation of Hamel coefficients and eliminates all the difficulties associated with the determination of these quantities. The equations of motion were solved with Wolfram Mathematica. In order to make the unicyclist-imitating part of the model closer to reality, it was based on the main principles of biomechanics. The impact of a pneumatic tyre was investigated with the Pacejka Magic Formula scheme including the experimental determination of the stiffness coefficient.

The aim of control is to maintain the unicycle–unicyclist system in an unstable equilibrium around the given angular position. The control system based on the LQ Regulator was applied in Wolfram Mathematica. In order to examine the legitimacy of the model experimental validation, the 3-D motion capture using the OptiTrack Motive: Body software together with high-speed cameras was arranged. A description of the unicycle–unicyclist dynamical model, simulation results and experimental validation of the system are presented in the work.

This book is organized as follows: designs of different types of unicycles are described in Chap. 1, a model which represents the unicyclist riding the unicycle straight ahead is derived in Chap. 2, whereas numerical and experimental validation of the model is shown in Chap. 3. Finally, concluding remarks are drawn in Chap. 4.

We would like to acknowledge the helpful suggestions and discussions with Jarosław Strzałko, Juliusz Grabski and Jerzy Wojewoda.

Finally, we would like to acknowledge the help and support of our families and friends.

Łódź, Poland — Michał Niełaczny
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April 2018

Contents

Notations

a_{ij}	Element of the matrix $\boldsymbol{A}$
$\boldsymbol{A}$	Matrix transforming generalized velocities into quasi-velocities
$\mathbb{A}$	State matrix
$\boldsymbol{B}$	Inverse matrix to the matrix $\boldsymbol{A}$
b_{nj}	Element of the matrix $\boldsymbol{B}$
$\mathbb{B}$	Input matrix
CP	Contact point of the unicycle wheel and ground
C_r, C_l	Points, ends of each crank (left, right)
$\boldsymbol{D}$	Derivative of the matrix $\boldsymbol{A}$ with respect to the vector $\boldsymbol{q}$
d	Damping coefficient
E_{ju}	Coefficient in the Pacejka Magic Formula
F	Force in the Pacejka Magic Formula
g	Gravitational acceleration
g_j	Control gain of the j-th state variable
$\boldsymbol{G}$	Three-dimensional matrix $(k \times k \times k)$ of Hamel symbols
H	Point, centre of mass of the wheel
H_r, H_l	Points, projection of H, on each leg motion plane
i	Number of the link
I_{ix}	Mass moment of inertia of the i-th link related to the axis x
I_{iy}	Mass moment of inertia of the i-th link related to the axis y
I_{iz}	Mass moment of inertia of the i-th link related to the axis z
$\boldsymbol{I}_i$	Matrix of mass moments of inertia of the i-th link
$\mathbb{I}$	Control identity matrix
j	Next natural numbers, $j = (1, 2, 3\ldots)$
k_t	Unicycle tyre stiffness coefficient
k_r	Unicycle rim stiffness coefficient
K_4, K_6	Points, each knee
$\mathbb{K}$	Law that minimizes the value of the cost
l_i	Auxiliary length of the i-th link
L_i	Length of the i-th link

m_i	Auxiliary mass of the i-th link
M_i	Mass of the i-th link
$\boldsymbol{M}_i$	Identity matrix of mass of the i-th link
O	Point, origin of the fixed frame $Oxyz$
P_4, P_6	Points, centre of mass of each pedal
q	Generalized coordinate
Q	External force
$\mathbb{Q}$	Matrix defines weights on states
r_i	Auxiliary radius of the i-th link
R_i	Radius of the i-th link
$\boldsymbol{R}_{\alpha i}$	Coordinates transformation matrix—rotation by the angle α_i around the axis z_{0i}
$\boldsymbol{R}_{\beta i}$	Coordinates transformation matrix—rotation by the angle β_i around the axis x_{0i}
$\boldsymbol{R}_{\gamma i}$	Coordinates transformation matrix—rotation by the angle γ_i around the axis z_i
$\boldsymbol{R}_i$	Coordinates transformation matrix of the i-th link from $\nabla\xi_i\eta_i\zeta_i$ to $Oxyz$
$\mathbb{R}$	Matrix defines weights on a control input in the cost function
S	Contact point of the saddle with the body
S_v	Offset in the Pacejka MF
t	Time
T	Kinetic energy
T^*	Kinetic energy expressed by quasi-velocities and generalized coordinates
u	Input quantity in the Pacejka MF
$\boldsymbol{u}$	Feedback control
v	Linear velocity
$\boldsymbol{v}$	Vector of linear velocities of the dimension $(k \times 1)$
V	Potential energy
w	Quasi-velocity
$\boldsymbol{w}$	Vector of quasi-velocities of the dimension $(k \times 1)$
$\boldsymbol{x}$	State variable
α_i	Angle of rotation around the axis z_{1i}
β_i	Angle of rotation around the axis x_{2i}
γ_i	Angle of rotation around the axis z_{3i}
$\left[\gamma^i_{nj}\right]$	Hamel coefficient
μ, χ, δ, v	Angles to derive general coordinates directly related to the leg
Λ_{ij}	Coefficients for the leg motion, shorten notation
π	Number pi
ϖ	Correlation coefficient
$\boldsymbol{\rho}$	Variable
ρ_i	Density of the i-th link
φ	Relative angle of rotation of the crank with respect to the frame
ψ	Quasi-coordinate

$\boldsymbol{\Psi}$	Load related to the quasi-coordinate
ω	Angular velocity
$\boldsymbol{\omega}$	Vector of angular velocities of the dimension $(k \times 1)$
$Oxyz$	Main fixed inertial frame, attached in the point O
∇	Any point, the origin of the frame
$\nabla x'y'z'$	Moving non-inertial frame, parallel to $Oxyz$, attached in the point ∇
$\nabla x_{0i}y_{0i}z_{0i}$	Moving non-inertial frame, related to the i-th link, attached in the point ∇
$\nabla x_i y_i z_i$	Moving non-inertial frame, related to the i-th link, attached in the point ∇
$\nabla \xi_i \eta_i \zeta_i$	Moving non-inertial frame, pegged to the i-th link, attached in the point ∇