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# Out-of-Equilibrium Physics of Correlated Electron Systems

 Springer

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# Preface

This volume “Out-of-Equilibrium Physics of Correlated Electron Systems” covers different theoretical aspects of the physics of non-equilibrium, ranging from the analytical to numerical and computational ones. It starts with the basic theory of master equations versus Keldysh Green’s function formalism for correlated quantum systems out-of-equilibrium, passing on to the Gutzwiller variational method and non-equilibrium mean-field theory for superconductors, concluding with the dynamical mean-field theory of correlated electron models. The book offers a comprehensive overview of the recent advancements in the theoretical investigations of strongly correlated systems out-of-equilibrium with specific examples.

The book contains the notes of the lectures delivered at the “Twentieth Training Course in the Physics of Strongly Correlated Systems” held in Vietri sul Mare (Salerno, Italy) in October 2016. The course consisted of four lectures every morning, held by Profs. Enrico Arrigoni, Massimo Capone, Martin Eckstein, Stefan Kaiser, and afternoon activities (seminars delivered by the junior researchers, solving of specific problems, roundtable on hot topics, hands-on training on relevant numerical issues) aimed principally at promoting discussions between the attendees and the lecturers. The outcome of this type of course was a significant interchange of ideas among the participants thanks to both the enlightening morning lectures and the long afternoon sessions devoted to discussions.

The book has both introductory and pedagogical aspects that could be very useful for researchers entering the field of non-equilibrium physics of correlated electron systems and the readers will strongly benefit from the different overviews on the topic.

We wish to acknowledge the support of those institutions that made the course possible. The main sponsors of the event were the Department of Physics “E.R. Caianiello”—University of Salerno and the International Institute for Advanced Scientific Studies “E.R. Caianiello.”

Fisciano, Italy  
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# Contents

<b>1</b>	<b>Introduction to the Volume</b> . . . . .	1
	Roberta Citro and Ferdinando Mancini	
1.1	Introduction . . . . .	1
	References . . . . .	3
<b>2</b>	<b>Towards the Understanding of Superconductors and Correlated Materials out of Equilibrium: Mean Field Approaches</b> . . . . .	5
	Massimo Capone and Carla Lupo	
2.1	Motivation and Introduction . . . . .	5
2.2	The Gutzwiller Variational Method . . . . .	8
2.2.1	The Time-Independent Gutzwiller Approximation . . . . .	9
2.2.2	An Explicit Example: The Single Band Hubbard Model . . . . .	13
2.2.3	Time-Dependent Gutzwiller Approximation . . . . .	16
2.2.4	Quantum Quench of the Interaction in the Hubbard Model . . . . .	19
2.3	Dynamical Mean-Field Theory . . . . .	20
2.3.1	The Hubbard Model . . . . .	24
2.3.2	Antiferromagnetism in the Hubbard Model . . . . .	26
2.3.3	The Attractive Hubbard Model . . . . .	29
2.4	Quantum Quenches for the Hubbard Model . . . . .	33
2.4.1	Quench Dynamics in the Gutzwiller Approximation . . . . .	33
2.4.2	Dynamical Phase Transition Within DMFT . . . . .	38
2.4.3	From Adiabatic Switching to Quantum Quenches: Ramping up the Interaction . . . . .	38
2.4.4	Interaction Quench in the Antiferromagnetic Hubbard Model . . . . .	40
2.5	Non-equilibrium Superconductors: Mean-Field Theory . . . . .	41
2.5.1	Anderson Pseudospin Representation . . . . .	46
2.5.2	Equations of Motions for the Pairing Amplitudes . . . . .	48

2.5.3	Dynamics After a Quantum Quench . . . . .	49
2.5.4	The Phase Diagram of s-wave Superconductors After a Quantum Quench . . . . .	50
2.5.5	Comparison Between s-wave and d-wave Superconductors . . . . .	52
2.5.6	Dynamics After a Small Perturbation . . . . .	55
	References . . . . .	56
<b>3</b>	<b>From the Keldysh Formalism to Non-equilibrium Dynamical Mean-Field Theory . . . . .</b>	<b>61</b>
	Martin Eckstein	
3.1	Introduction . . . . .	61
3.2	Green's Functions and Many-Body Systems Out of Equilibrium . . . . .	64
3.2.1	The Thermal Equilibrium State . . . . .	64
3.2.2	Green's Functions and Electronic Structure . . . . .	68
3.2.3	Probabilistic Interpretation of Real-Time Green's Functions . . . . .	70
3.3	The Keldysh Formalism . . . . .	72
3.3.1	The Time-Evolution Operator . . . . .	72
3.3.2	Time-Dependent Expectation Values and the Keldysh Contour . . . . .	73
3.3.3	Contour-Ordered Green's Functions . . . . .	75
3.3.4	Noninteracting Green's Function . . . . .	78
3.3.5	The Two-Time Self-energy . . . . .	80
3.3.6	Self-energy of the Open Quantum System . . . . .	81
3.3.7	Diagrammatic Perturbation Theory . . . . .	82
3.4	The Dyson Equation . . . . .	85
3.4.1	Langreth Rules . . . . .	86
3.4.2	Kadanoff-Baym Equations . . . . .	88
3.4.3	Steady-State Formalism . . . . .	91
3.5	Nonequilibrium Dynamical-Mean-Field Theory . . . . .	92
3.5.1	The Dynamical Mean-Field Formalism . . . . .	92
3.5.2	Bethe Lattice . . . . .	96
3.5.3	Numerical Implementation and Impurity Solvers . . . . .	97
3.6	Photo-Doped Mott Insulators . . . . .	99
3.6.1	The Mott Transition in DMFT . . . . .	99
3.6.2	Paramagnetic Phase - Dynamics of Photo-Excited Doublons . . . . .	102
3.6.3	Antiferromagnetic Case . . . . .	107
3.6.4	The Buildup of the Fermi Liquid . . . . .	110

3.7	Concluding Remarks . . . . .	114
	References . . . . .	114
<b>4</b>	<b>Master Equations Versus Keldysh Green's Functions for Correlated Quantum Systems Out of Equilibrium</b> . . . . .	<b>121</b>
	Enrico Arrigoni and Antonius Dorda	
4.1	Introduction . . . . .	121
4.2	Master Equations . . . . .	125
4.3	Density Matrix . . . . .	127
	4.3.1 Time Dependence . . . . .	128
	4.3.2 Reduced Density Matrix . . . . .	130
4.4	Lindblad Equation . . . . .	133
	4.4.1 Heuristic Derivation . . . . .	134
	4.4.2 Solution of the Lindblad Equation by Exact Diagonalization . . . . .	136
	4.4.3 Fermionic Model Described by the Lindblad Equation . . . . .	137
	4.4.4 Microscopic Derivation of the Lindblad Equation . . . . .	137
	4.4.5 Derivation for a Fermionic System-Reservoir Setup . . . . .	146
4.5	Superfermion Representation . . . . .	150
	4.5.1 Representation of the Lindblad Equation . . . . .	153
4.6	Correlation Functions and Quantum Regression Theorem . . . . .	157
	4.6.1 Superfermion Representation . . . . .	159
	4.6.2 Fermionic Operators . . . . .	160
4.7	Nonequilibrium Green's Functions . . . . .	161
	4.7.1 Anderson Impurity Model . . . . .	163
4.8	Nonequilibrium Impurity Problems . . . . .	166
	4.8.1 Buffer Layer Approach . . . . .	168
	4.8.2 Finite Size Lindblad Impurity Problem . . . . .	171
4.9	Auxiliary Master Equation Approach . . . . .	172
	4.9.1 Evaluation of Steady State Green's Functions . . . . .	173
	References . . . . .	184
	<b>Index</b> . . . . .	<b>189</b>



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