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Michał Piórek

# Analysis of Chaotic Behavior in Non-linear Dynamical Systems

## Models and Algorithms for Quaternions

Springer

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*To my Wife and my Family*

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