Mathematics in Mind

The monographs and occasional textbooks published in this series tap directly into the kinds of themes, research findings, and general professional activities of the Fields Cognitive Science Network, which brings together mathematicians, philosophers, and cognitive scientists to explore the question of the nature of mathematics and how it is learned from various interdisciplinary angles.

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The question of the nature of mathematics is actually an empirical question that can best be investigated with various disciplinary tools, involving diverse types of hypotheses, testing procedures, and derived theoretical interpretations. This series aims to address questions of mathematics as a unique type of human conceptual system versus sharing neural systems with other faculties, whether it is a species-specific trait or exists in some other form in other species, what structures (if any) are shared by mathematics, language, and more.

Data and new results related to such questions are being collected and published in various peer-reviewed academic journals. Amongst other things, data and results have profound implications for the teaching and learning of mathematics. The objective is based on the premise that mathematics, like language, is inherently interpretive and explorative at once. In this sense, the inherent goal is a hermeneutical one, attempting to explore and understand a phenomenon—mathematics—from as many scientific and humanistic angles as possible.

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Ahmes' Legacy

Puzzles and the Mathematical Mind

Springer

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Preface

A good puzzle, like virtue, is its own reward.—Henry E. Dudeney (1857–1930)

Puzzles emerged at the dawn of history, and perhaps even before that (Olivastro 1993), putting on display the powers of the human imagination to grasp truths in its own playful way. There is no culture without riddles, no era of time without clever conundrums that are intended to defy logic, and no faculty of mind, from language to visual perception, that has not produced its own ludic artifacts. In mathematics, puzzles have often played critical roles in the history of the discipline, above all else as miniature investigative models of inherent principles or patterns of hidden structure, often leading to significant discoveries. For instance, Alcuin's River Crossing Puzzle or Euler's Königsberg Bridges Puzzle contained, respectively, the blueprints for combinatorics and graph theory. Even one of the oldest mathematics texts, the Egyptian *Ahmes Papyrus*, which dates back to before 1650 BCE, turns out to be essentially a collection of puzzles that were designed to illustrate specific mathematical ideas and to emphasize the power and simple beauty of mathematics—the same beauty of which the Pythagoreans spoke and which they saw reflected in music and in the movements of cosmic bodies alike.

This book will attempt to argue that the origins of some, if not many, mathematical concepts originate in the form of suggestive puzzles. Although this is well known on the part of mathematicians, and even though the branch of "recreational mathematics" provides a specific locus in which to examine individual puzzles and their theoretical implications, there are few overall general psychological discussions of the significance of puzzles as revelatory manifestations of how the mathematical mind works, as far as I can tell. I have not designed this book as a rigorous treatment of puzzles within the domain of recreational mathematicians, but rather as an excursion into the relation of puzzles to mathematical discovery, and what this relation tells us about the brain. No particular technical knowledge is thus required, since I will discuss each and every puzzle in a general way. My objective throughout is to show how some of the classic puzzles of mathematical history are essentially imaginative explorations of quantity and space in ludic form. Needless to say, so many ingenious puzzles have been invented that it would be brazenly presumptuous to claim that I have chosen the most important or paradigmatic ones here. Moreover, most have already been studied in depth as sources shaping significant parts of mathematical history. My aim is to take a plausible look, so to say, inside the mind of the puzzler, both as maker and solver, in order to trace the source of puzzles within the mind. This book is not a historiography of puzzles, which I attempted to treat schematically elsewhere (Danesi 2002), although the histories of some puzzle genres will be discussed whenever they are relevant. Rather, I have written it to convey the intriguing psychological story that puzzles tell us about the mathematical imagination.

As mentioned, in no way do I intend to imply that mathematicians are unaware of the cognitive value of studying puzzles. Indeed, they discuss them, write about them, and research them in meaningful ways. So, this book is really an overview, a summing-up, or synopsis, of the role of puzzles in the history of mathematics. I have discussed the ideas in this book with students in a course I have been teaching for over a decade at Victoria College at the University of Toronto called "Puzzles, Discovery, and the Human Imagination." I am very grateful to them for all their insights over the years. I am also grateful to Elizabeth Loew at Springer for giving me the opportunity to put my ideas down in writing, and to Dahlia Fisch for her expert editorial advice and encouragement. Any infelicities that this book contains are my sole responsibility. I sincerely hope that mathematicians and general readers alike will glean something from it that might have escaped their attention, and thus provide an "interesting" perspective on mathematical discovery. I truly believe that the systematic study of puzzles is as important as the study of any human construct or mode of creative expression. As the great modern-day mathematician, David Hilbert, once observed, "Mathematics is a game played according to certain rules with meaningless marks on paper" (cited in Pillis and Rose 1988). The creative source of those marks is often to be found in math games themselves.

Toronto, ON Marcel Danesi
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