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Accretion Flows in Astrophysics

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Cover illustration: Visualization of a twisted disc according to a solution by Scheuer and Feiler (1996). The action of viscosity on a differentially precessing disc ensures that the inner portions of the disc are aligned with the angular momentum of the black hole; further out, the disc is warped where the transition from aligned to misaligned disc occurs (Bardeen and Petterson 1975), Copyright Galina Lipunova. Background: Astronomy Blue Bright Clouds, Photographer: Albin Berlin, https://www.pexels.com/photo/astronomy-blue-bright-clouds-340901/, Creative Commons Zero (CC0) license

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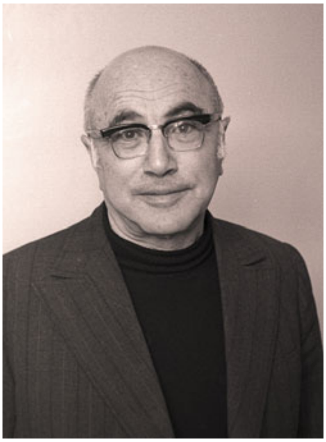
Preface

The word "accretion" has a Latin origin (*accretio*) and means an augmentation or increment of an initial amount by the addition of new portions. Astronomers use this term to describe the falling of diffused matter onto a center of gravity. Accretion onto compact stellar objects, for example, neutron stars and black holes, is accompanied by an enormous output of energy. In the 1970s, the study of such processes became of special importance. It was the time when the American UHURU satellite discovered X-ray emission from accreting black holes and neutron stars in binary stellar systems. Some time earlier, in the end of the 1960s, when I was a graduate student at the physics faculty of the Moscow State University (MSU), my scientific advisor academician Ya. B. Zeldovich suggested to me to calculate the structure and radiation spectra of the shock wave arising when gas accretes onto a neutron star. The choice of this particular scientific problem was triggered by the following circumstances.

In 1962, a group of American scientists led by Prof. Riccardo Giacconi discovered the first X-ray sources. Before that, astronomers had known only one X-ray source of extraterrestrial origin, namely, the solar corona. The coronal gas, heated to a million degrees by some then unknown mechanism, was known to produce X-ray emission. The luminosity of the solar corona in X-rays is approximately one millionth of the optical luminosity of the Sun (4×10^{33} erg/s). It was thus natural to assume that other stars are also surrounded by hot coronae. However, simple calculations showed that detectors available at that time could not detect coronae even around the nearest stars located at distances of a few parsecs.

Nevertheless, astronomers tried to detect X-ray radiation from the Moon! The Moon has no atmosphere, but perhaps some radiation could be produced by fluorescence as the Moon's surface is illuminated by X-rays from the Sun. To investigate this, precisely at midnight of June 18, 1962, when a full moon was shining, the Aerobee rocket was launched. It reached a height of 225 km. Its flight continued for 350 s and was quite successful: two of the three Geiger counters, with large surface and good sensitivity in the range 1.5–6 keV, were operating during the flight. In this energy range, the Earth's atmosphere is totally opaque. Suddenly, instead of X-ray radiation from the Moon, a bright and before unknown source was discovered, which was far beyond the solar system in the direction of the constellation Scorpius. It was named Sco X-1.

In the following years, new rocket flights brought more and more discoveries of new X-ray sources. Gradually, a sky map covered with X-ray sources of different nature was created. The first sources got their names according to their location in the night sky (Cyg X-1, Cyg X-2, Her X-1, Cen X-3, and so on). Later it was revealed that their X-ray luminosities were thousands or even tens of thousand times stronger than the Sun's luminosity in visual light. The epoch of X-ray astronomy, an epoch of stunning discoveries in the universe, began.



Ya. B. Zeldovich. Credit: Photo-Archive of the Sternberg Astronomical Institute

According to simple estimates made by Ya. B. Zeldovich himself, the shock wave arising when the gas surrounding a neutron star falls onto its surface should produce radiation primarily in the X-ray range. My goal was to carry out a full calculation and investigate the process in detail. The main difficulty was connected to the following property: the mean free path of a falling particle near the surface of the neutron star is much greater (tens of times) than the characteristic scale of interaction between matter and radiation. In many such problems, it is not necessary to calculate the structure of the shock wave: it is sufficient to specify the changes in density, pressure, temperature, and other physical parameters depending on the velocity and the adiabatic index of the falling gas. In my problem, the density, temperature, and other parameters depended on the energy release in the braking zone. Moreover, plasma oscillations may arise in this zone. To describe these, the use of kinetic plasma equations is required rather than ordinary hydrodynamics. In the end, however, I managed to show that shock wave emission spectra from accreting neutron stars could explain the observational data obtained with the recently launched instruments.

The first identifications of cosmic X-ray sources with their optical counterparts appeared in the 1960s, allowing a determination of their luminosities and distances to them. It became clear that the large luminosities of accreting neutron stars could be provided only in close binary systems with mass flowing from the stellar component to the neutron star.

When I was a student of astronomy, I attended a course in astrophysics given by the director of the Sternberg Astronomical Institute, Prof. D. Ya. Martynov. In his lectures, he paid special attention to the processes of mass exchange in binary stellar systems through the inner Lagrangian point, and explained how due to the relative orbital motion of the components, a stream of gas forms a disc-like envelope around one of the stars. So, I decided to place a neutron star or even a black hole as an accreting component in a binary system and found that in this case, a new type of accretion (namely, disc accretion!) is possible since the matter, which falls onto such a powerful center of gravity as a neutron star or a black hole, possesses a large angular momentum that prevents it from falling radially inwards. In a first approximation, matter in the disc moves along nearly Keplerian orbits. Slow radial movement of the disc matter toward the center of gravity accompanied by a large energy output (disc accretion) can only be triggered by exchange of angular momentum between adjacent layers of the differentially rotating disc. The reason for such exchange could be turbulence and/or magnetic fields.

In 1969, the article with the calculation and description of the shock wave structure was published in "Astronomicheskii Zhurnal" and became my diploma. This year, I became a postgraduate student at the physics faculty of the MSU. Academician Ya. B. Zeldovich became my scientific advisor.

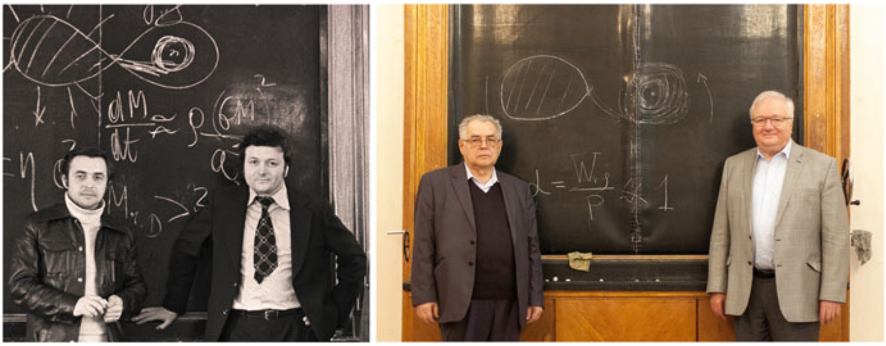
As a postgraduate student, I continued to study the structure and spectra of accretion discs that form around accreting neutron stars and black holes in close binaries due to mass flow from the surface of an optical star.

D. Ya. Martynov. Credit: Photo-Archive of the Sternberg Astronomical Institute

The foundations of the theory of disc accretion were published, also in "Astronomicheskii Zhurnal", in 1972. The main part of the work was done in collaboration with R. A. Sunyaev. Together we developed the so-called standard model of disc accretion. The work was presented at the 55th symposium of the International Astronomical Union in Madrid in 1972 (Shakura and Sunyaev 1973b). It was there that the first observational results from the UHURU satellite were presented and the first theoretical models of compact X-ray sources in stellar binaries discovered by UHURU were reported. Our report in Madrid was an introduction to a highly influential article published in "Astronomy and Astrophysics" in 1973 (Shakura and Sunyaev 1973a). On the basis of this article, I. D. Novikov and K. S. Thorne calculated the relativistic corrections required by general relativity (Novikov and Thorne 1973).

The pioneering work made together with R. A. Sunyaev is still topical today. According to the NASA ADS data system, the number of references to this article exceeds 8400 (as of April of 2018). It is our great pleasure to present to you this book covering some of the most principal and important areas of modern theory of disc and quasi-spherical accretion onto black holes and magnetized neutron stars.

In Chap. 1, the authors (G. V. Lipunova, K. L. Malanchev, and N. Shakura) present the equations of disc accretion in the framework of the standard model, the basics of the phenomenological theory of turbulent viscosity, and the properties of thin accretion discs and their structure along the radial and vertical directions. The



N. Shakura and R. A. Sunyaev in 1973 and 2017

authors describe analytical solutions to the basic equation of evolution of a non-stationary viscous accretion disc, in the case of infinitely large discs and for discs in binary systems enclosed within their Roche lobes. It is shown how the characteristic time scale of variability in non-stationary disc accretion allows us to determine the level of developed turbulence in accretion discs. A method for a joint numerical solution of the evolution equation and the equations of vertical disc structure is presented.

Chapter 2 (by N. Shakura) is devoted to motion of particles along spherical geodesics around rotating black holes. Such motion is possible if the plane of the outer parts of the accretion disc is tilted toward the equatorial plane of the rotating black hole. A study of this motion is necessary for understanding the structure of a warped disc. This chapter uses a special approach to determine how the quantities, which are measured in the local frame of a fiducial observer in the axially symmetric gravitational field, are related to each other. This approach allows us to better understand the basic principles for measuring physical quantities in GR. These basic principles, which are systematically presented in the next chapter, are required for a more comprehensive understanding of the structure of relativistic accretion discs.

Chapter 3 (by V. V. Zhuravlev) presents a self-consistent model of a standard relativistic accreion disc. The disc is aligned with the equatorial plane of a rotating black hole, and calculation is performed taking full account of relativistic effects. In the first half of the chapter, the author describes in detail how relativistic corrections to the disc structure are deduced using a tetrad basis that is carried by an observer comoving with the rotating matter. Further, using the basic simplifying assumptions of the standard accretion disc model, the relativistic hydrodynamic equations are projected onto the tetrad basis. After that, the author presents an explicit relativistic generalization of radial profiles of the viscous stress and the energy flux from the disc surface.

In Chap. 4, V. V. Zhuravlev outlines the theory of twisted relativistic accretion discs. A warped disc forms around a rotating black hole if the outer parts of the disc are not aligned with the black hole's equatorial plane. The author derives the equations describing the evolution of the shape of a twisted disc and the perturbations of density and velocity necessarily arising in such a disc. This is done under some simplifying assumptions (namely, a small aspect ratio of the disc, slow rotation of the black hole, and a small tilt angle of the disc rings with respect to the black hole equatorial plane), nevertheless including all general relativity effects. The author further presents an analysis of particular regimes of nonstationary twist dynamics (the wave and diffusion regimes), both in the framework of Newtonian dynamics and taking into account Einstein's relativistic precession. At the end of the chapter, a calculation of the shape of a stationary relativistic twisted accretion disc for different values of free parameters of the model is presented.

In Chap. 5, the authors (P. K. Abolmasov, N. Shakura, and A. A. Chashkina) examine the structure of accretion discs in distant quasars from the point of view of the spatial information obtained with the help of quasar microlensing. This exotic effect appears when strong lensing by a foreground galaxy is accompanied by microlensing on individual stars in it. The authors of this chapter aim to give a general introduction to QSO microlensing and to show the opportunities of the method, providing a review of the recent results in this area. It is also shown that the typical variability of the radiation (observed in different spectral ranges) caused by microlensing allows us to study the structure of both subcritical and supercritical (super-Eddington) accretion discs. The latter are characterized by outflow of matter from the inner parts of the disc due to strong radiation pressure. As a consequence, a quasi-spherical envelope forms with a radius determined by processes of scattering by free electrons. This radius has different, presumably weaker, dependence on wavelength, whereas the effective radius of the standard subcritical disc is proportional to the wavelength as $r \sim \lambda^{4/3}$.

Chapter 6 (by V. V. Zhuravlev and D. N. Razdoburdin) is focused on the study of transient growth of small perturbations in spectrally stable rotating shear flows, in particular, those with a Keplerian profile of angular velocity. The mechanism of perturbation growth is discussed in the simplest model of local two-dimensional adiabatic perturbations in a spatially homogeneous flow. Furthermore, special emphasis is placed on mathematical methods that make it possible to perform a rigorous analysis of transient dynamics in disc models of various sophistication. The transient growth of perturbations seems to be capable of transferring energy from a background flow to perturbations in a homogeneous Keplerian flow (in the absence of a magnetic field). Without this energy transport, the emergence of turbulence and/or enhanced angular momentum flux towards the disc outskirts would not be possible.

In Chap. 7, the authors (N. Shakura, K. A. Postnov, A. Yu. Kochetkova, and L. Hjalmarsdotter) examine the theoretical model of quasi-spherical subsonic accretion onto slowly rotating magnetized neutron stars. In this case, the accreting matter slowly, with subsonic velocity, settles onto the rotating magnetosphere of the neutron star, forming an elongated quasi-spherical envelope. The angular momentum transfer in the envelope is effected through large-scale convective motions, implying that the differential rotation law in envelopes above magnetospheres of actual X-ray pulsars corresponds to an approximately isomomentum distribution. The accretion rate in the envelope depends on the ability of the plasma to penetrate into the magnetosphere due to the Rayleigh–Taylor instability, if cooling processes are taken into account. Subsonic infall of matter may occur at moderate X-ray luminosities corresponding to accretion rates of $\dot{M} \lesssim 4 \times 10^{16}$ g/s. In the case of higher accretion rates, a region of free-falling matter arises in the flow above the magnetosphere due to fast Compton cooling, making accretion highly nonstationary. One can determine the basic parameters of the model and estimate the magnetic field of the neutron star when observing acceleration and slowdown in the rotation periods of equilibrium X-ray pulsars with known orbital periods, such as GX 301-2 and Vela X-1, in which quasi-spherical accretion from the stellar wind occurs. It is possible to estimate the velocity of the stellar wind emitted by the optical counterpart of an equilibrium pulsar in a binary, without conducting complex spectroscopic measurements, if an independently measured magnetic field is known for the neutron star. There is a maximal possible value for the slowdown rate of the neutron star for accretion onto a nonequilibrium pulsar. Examples of such pulsars are GX 1+4, SXP 1062 and 4U 2206+54. Knowing the slowdown rate of the rotation of such a pulsar and its X-ray luminosity, we may estimate a lower limit on the magnetic field of the neutron star, which always turns out to be close to the standard value and corresponds to the observed cyclotron peculiarities in measured spectra. The model explains why rotation in nonequilibrium pulsars accelerates and slows down on long timescales and why the pulsar frequency varies on short timescales. In different binaries, these variations may display either a correlation or an anticorrelation with the observed fluctuations in X-ray flux.

The authors of Chap. 8 (N. Shakura and K. A. Postnov) examine the conditions under which the Velikhov–Chandrasekhar magneto-rotational instability (MRI) in ideal and nonideal plasma may arise. In the presence of magnetic fields, this instability arises in an axially symmetric hydrodynamic flow if the angular velocity of the flow decreases outwards, whereas the angular momentum increases. The growth rate of MRI decreases if the magnetic field becomes stronger; there is a critical value of the magnetic field, above which the exponential growth gives way to oscillations. The influence of viscosity and electrical conductivity of the plasma on the development of MRI is studied. The limiting values (lower limits) of ion mean free paths, for which MRI is still possible in thin discs obeying Kepler's law of rotation, are obtained.

On the other hand, the authors show that the MHD mode becomes stabilized in a hydrodynamic flow, which is unstable according to the Rayleigh criterion (the angular momentum decreases outwards), for small perturbation wavelengths.

Many excellent books have been published about accretion in astrophysics. We would like to mention, in particular, *Black-Hole Accretion Disks* by S. Kato, J. Fukue, and S. Mineshige; *Oscillations of Disks* by S. Kato; and *Accretion Power in Astrophysics* by J. Frank, A. King, and D. Raine. We hope that the reader will find the present volume useful as well.

Moscow, Russia Nikolay Shakura
April 2018

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