

Studies in Computational Intelligence

Volume 773

Series editor

Janusz Kacprzyk, Polish Academy of Sciences, Warsaw, Poland
e-mail: kacprzyk@ibspan.waw.pl

The series “Studies in Computational Intelligence” (SCI) publishes new developments and advances in the various areas of computational intelligence—quickly and with a high quality. The intent is to cover the theory, applications, and design methods of computational intelligence, as embedded in the fields of engineering, computer science, physics and life sciences, as well as the methodologies behind them. The series contains monographs, lecture notes and edited volumes in computational intelligence spanning the areas of neural networks, connectionist systems, genetic algorithms, evolutionary computation, artificial intelligence, cellular automata, self-organizing systems, soft computing, fuzzy systems, and hybrid intelligent systems. Of particular value to both the contributors and the readership are the short publication timeframe and the world-wide distribution, which enable both wide and rapid dissemination of research output.

More information about this series at <http://www.springer.com/series/7092>

Andrew Pownuk · Vladik Kreinovich

Combining Interval, Probabilistic, and Other Types of Uncertainty in Engineering Applications

Andrew Pownuk
Computational Science Program
University of Texas at El Paso
El Paso, TX
USA

Vladik Kreinovich
Computational Science Program
University of Texas at El Paso
El Paso, TX
USA

ISSN 1860-949X ISSN 1860-9503 (electronic)
Studies in Computational Intelligence
ISBN 978-3-319-91025-3 ISBN 978-3-319-91026-0 (eBook)
<https://doi.org/10.1007/978-3-319-91026-0>

Library of Congress Control Number: 2018940421

© Springer International Publishing AG, part of Springer Nature 2018

This work is subject to copyright. All rights are reserved by the Publisher, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilms or in any other physical way, and transmission or information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed.

The use of general descriptive names, registered names, trademarks, service marks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

The publisher, the authors and the editors are safe to assume that the advice and information in this book are believed to be true and accurate at the date of publication. Neither the publisher nor the authors or the editors give a warranty, express or implied, with respect to the material contained herein or for any errors or omissions that may have been made. The publisher remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

Printed on acid-free paper

This Springer imprint is published by the registered company Springer International Publishing AG part of Springer Nature
The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

Contents

1 Introduction	1
1.1 Need for Data Processing	1
1.2 Need to Take Uncertainty Into Account When Processing Data	2
1.3 How to Gauge the Accuracy of the Estimates \tilde{x}_i	2
1.4 Measurement and Estimation Inaccuracies Are Usually Small	3
1.5 How to Estimate Partial Derivatives c_i	4
1.6 Existing Methods for Computing the Probabilistic Uncertainty: Linearization Case	5
1.7 Existing Methods for Computing the Interval Range: Linearization Case	6
1.8 Existing Methods for Estimating Fuzzy Uncertainty: Linearization Case	10
1.9 Open Problems and What We Do in This Book	11
2 How to Get More Accurate Estimates	13
2.1 In System Identification, Interval Estimates Can Lead to Much Better Accuracy than the Traditional Statistical Ones: General Algorithm and Case Study	13
2.1.1 Formulation of the Problem	14
2.1.2 System Identification Under Interval Uncertainty: General Algorithm	16
2.1.3 System Identification Under Interval Uncertainty: Simplest Case of Linear Dependence on One Variable	20
2.1.4 Case Study	23
2.1.5 Conclusions	24
2.2 Which Value \tilde{x} Best Represents a Sample x_1, \dots, x_n : Utility-Based Approach Under Interval Uncertainty	25
2.2.1 Which Value \tilde{x} Best Represents a Sample x_1, \dots, x_n : Case of Exact Estimates	25

2.2.2	Case of Interval Uncertainty: Formulation of the Problem	27
2.2.3	Analysis of the Problem	27
2.2.4	Resulting Algorithm	29
2.3	How to Get More Accurate Estimates – By Properly Taking Model Inaccuracy into Account	30
2.3.1	What if We Take into Account Model Inaccuracy	30
2.3.2	How to Get Better Estimates	32
2.3.3	Can We Further Improve the Accuracy?	38
2.4	How to Gauge the Accuracy of Fuzzy Control Recommendations: A Simple Idea	40
2.4.1	Formulation of the Problem	40
2.4.2	Main Idea	41
2.4.3	But What Should We Do in the Interval-Valued Fuzzy Case?	42
3	How to Speed Up Computations	45
3.1	How to Speed Up Processing of Fuzzy Data	45
3.1.1	Cases for Which a Speedup Is Possible: A Description	46
3.1.2	Main Idea of This Section	48
3.2	Fuzzy Data Processing Beyond min t-Norm	49
3.2.1	Need for Fuzzy Data Processing	50
3.2.2	Possibility of Linearization	54
3.2.3	Efficient Fuzzy Data Processing for the min t-Norm: Reminder	57
3.2.4	Efficient Fuzzy Data Processing Beyond min t-Norm: the Main Result of This Section	59
3.2.5	Resulting Linear Time Algorithm for Fuzzy Data Processing Beyond min t-Norm	61
3.2.6	Conclusions	62
3.3	How to Speed Up Processing of Probabilistic Data	63
3.3.1	Need to Speed Up Data Processing Under Uncertainty: Formulation of the Problem	63
3.3.2	Analysis of the Problem and Our Idea	65
3.3.3	How to Approximate	67
3.3.4	Resulting Algorithm	69
3.3.5	Numerical Example	70
3.3.6	Non-uniform Distribution of α_j is Better	71
3.4	Hypothetical Quantum-Related Negative Probabilities Can Speed Up Uncertainty Propagation Algorithms	76
3.4.1	Introduction	76
3.4.2	Uncertainty Propagation: Reminder and Precise Formulation of the Problem	77

- 3.4.3 Existing Algorithms for Uncertainty Propagation and Their Limitations 82
- 3.4.4 Analysis of the Problem and the Resulting Negative-Probability-Based Fast Algorithm for Uncertainty Quantification 87
- 4 Towards a Better Understandability of Uncertainty-Estimating Algorithms 97**
 - 4.1 Case of Interval Uncertainty: Practical Need for Algebraic (Equality-Type) Solutions of Interval Equations and for Extended-Zero Solutions 97
 - 4.1.1 Practical Need for Solving Interval Systems of Equations: What Is Known 98
 - 4.1.2 Remaining Problem of How to Find the Set A Naturally Leads to Algebraic (Equality-Type) Solutions to Interval System of Equations 102
 - 4.1.3 What if the Interval System of Equations Does not Have an Algebraic (Equality-Type) Solution: A Justification for Enhanced-Zero Solutions 103
 - 4.2 Case of Interval Uncertainty: Explaining the Need for Non-realistic Monte-Carlo Simulations 106
 - 4.2.1 Problem: The Existing Monte-Carlo Method for Interval Uncertainty Is not Realistic 106
 - 4.2.2 Proof That Realistic Interval Monte-Carlo Techniques Are not Possible: Case of Independent Variables 107
 - 4.2.3 Proof That Realistic Interval Monte-Carlo Techniques Are not Possible: General Case 108
 - 4.2.4 Why Cauchy Distribution 114
 - 4.3 Case of Probabilistic Uncertainty: Why Mixture of Probability Distributions? 115
 - 4.3.1 Formulation of the Problem 116
 - 4.3.2 Main Result of This Section 116
 - 4.4 Case of Fuzzy Uncertainty: Every Sufficiently Complex Logic Is Multi-valued Already 119
 - 4.4.1 Formulation of the Problem: Bridging the Gap Between Fuzzy Logic and the Traditional 2-Valued Fuzzy Logic 119
 - 4.4.2 Fuzzy Logic – The Way It Is Typically Viewed as Drastically Different from the Traditional 2-Valued Logic 121
 - 4.4.3 There Is Already Multi-valuedness in the Traditional 2-Valued Fuzzy Logic: Known Results 121
 - 4.4.4 Application of 2-Valued Logic to Expert Knowledge Naturally Leads to a New Aspect of Multi-valuedness – An Aspect Similar to Fuzzy 122

4.5	General Case: Explaining Ubiquity of Linear Models	125
4.5.1	Formulation of the Problem	125
4.5.2	Analysis of the Problem: What Are Reasonable Property of an Interpolation	126
4.5.3	Main Result of This Section	128
4.6	General Case: Non-linear Effects	132
4.6.1	Compaction Meter Value (CMV) – An Empirical Measure of Pavement Stiffness	132
4.6.2	A Possible Theoretical Explanation of an Empirical Correlation Between CMV and Stiffness	133
5	How General Can We Go: What Is Computable and What Is not	137
5.1	Formulation of the Problem	137
5.2	What Is Computable: A Brief Reminder	138
5.3	What We Need to Compute: A Even Briefer Reminder	142
5.4	Simplest Case: A Single Random Variable.	142
5.5	What if We Only Have Partial Information About the Probability Distribution?	146
5.6	What to Do in a General (not Necessarily 1-D) Case	148
5.7	Proofs	150
5.8	Conclusions	154
6	Decision Making Under Uncertainty	157
6.1	Towards Decision Making Under Interval Uncertainty	157
6.1.1	Formulation of the Practical Problem	158
6.1.2	Formulation of the Problem in Precise Terms	158
6.1.3	Analysis of the Problem.	159
6.1.4	Explaining Why, In General, We Have $f_{x_i} \neq 0$	161
6.1.5	Analysis of the Problem (Continued)	162
6.1.6	Solution to the Problem	166
6.2	What Decision to Make In a Conflict Situation Under Interval Uncertainty: Efficient Algorithms for the Hurwicz Approach	167
6.2.1	Conflict Situations Under Interval Uncertainty: Formulation of the Problem and What Is Known So Far.	168
6.2.2	Conflict Situation Under Hurwicz-Type Interval Uncertainty: Analysis of the Problem	172
6.2.3	Algorithm for Solving Conflict Situation Under Hurwicz-Type Interval Uncertainty	174
6.2.4	Conclusion	176

- 6.3 Decision Making Under Probabilistic Uncertainty: Why Unexpectedly Positive Experiences Make Decision Makers More Optimistic 176
 - 6.3.1 Formulation of the Problem 177
 - 6.3.2 Formulating the Problem in Precise Terms 177
 - 6.3.3 Towards the Desired Explanation 178
- 6.4 Decision Making Under General Uncertainty 180
 - 6.4.1 Formulation of the Problem 180
 - 6.4.2 Analysis of the Problem 182
 - 6.4.3 Conclusions 187
- 7 Conclusions 191**
- References 193**
- Index 199**

Abstract

In many practical applications, we process measurement results and expert estimates. The measurements and expert estimates are never absolutely accurate, and their results are slightly different from the actual (unknown) values of the corresponding quantities. It is therefore desirable to analyze how these measurement and estimation inaccuracies affect the results of data processing.

There exist numerous methods for estimating the accuracy of the results of data processing under different models of measurement and estimation inaccuracies: probabilistic, interval, and fuzzy. To be useful in engineering applications, these methods should provide accurate estimate for the resulting uncertainty, should not take too much computation time, should be understandable to engineers, and should be sufficiently general to cover all kinds of uncertainty.

In this book, on several case studies, we show how we can achieve these four objectives. We show that we can get more accurate estimates, for example, by properly taking model inaccuracy into account. We show that we can speed up computations, e.g., by processing different types of uncertainty separately. We show that we can make uncertainty-estimating algorithms more understandable, e.g., by explaining the need for non-realistic Monte Carlo simulations. We also analyze how to make decisions under uncertainty and how general uncertainty-estimating algorithms can be.