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# Admissibility and Hyperbolicity

Springer

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# Preface

The main objective of this book is to give a fairly broad overview of the relation between admissibility and hyperbolicity, which goes back to the seminal work of Perron in 1930. In particular, we present the main directions of the theory and a few selected recent developments. We also give some applications of the relation between admissibility and hyperbolicity, as well as pointers for further topics that were already out of the scope of the book. Overall, the text is an invitation to the area and we hope that it may lead to further developments.

On purpose, we avoid discussing topics that are not directly related to admissibility or that are not applications of admissibility. This allows us to highlight the main aspects of the theory avoiding some technical complications that are not essential. Instead, we provide detailed references to the literature for those topics that are not discussed in the book. On the other hand, most of the material did not appear before in book form. Moreover, much of it was rewritten on purpose for our text.

The book is dedicated to researchers as well as graduate students specializing in differential equations and dynamical systems, with emphasis on hyperbolicity, who wish to have a comprehensive view of the relation between admissibility and hyperbolicity, as well as a working knowledge of its techniques. It can also be used as a basis for graduate courses on hyperbolicity with emphasis on admissibility.

The material is divided into three parts: the core of the theory is presented in Chapters 2–4, a few selected topics are discussed in Chapter 5, and some applications are given in Chapter 6. In the introduction we describe in a pragmatic manner the origins of the theory and we give a brief overview of the contents of the book.

In Chapter 2 we present the main results of the admissibility theory in the simpler case of exponential contractions, both for discrete and continuous time. This allows us to give a first introduction to the relation between hyperbolicity and admissibility without the technical complications caused by the existence of contraction and expansion in an exponential dichotomy.

Chapters 3 and 4 consider the general case of exponential dichotomies, respectively, for discrete and continuous time. In particular, we establish corresponding results using arguments that build on those in Chapter 2, although there are various technical difficulties that need to be overcome.

In Chapter 5 we consider various extensions of the results in the former chapters. Namely, we develop a general approach to the construction of admissible spaces, we present results dealing with a weaker form of admissibility that does not require the uniqueness of solutions, and we consider the more general notion of a nonuniform exponential behavior.

Finally, in Chapter 6 we describe various applications of the theory and in particular of the relation between admissibility and hyperbolicity to the robustness of an exponential dichotomy, to the characterization of hyperbolic sets in terms of admissibility, to the relation between shadowing and structural stability, and to the characterization of hyperbolicity in terms of Lyapunov sequences.

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