

## SPACE TECHNOLOGY LIBRARY

Published jointly by Microcosm Press and Springer

---

### The Space Technology Library Editorial Board

Managing Editor: **James R. Wertz** (Microcosm, Inc., El Segundo, CA)

Editorial Board:

- Roland Doré** (Professor and Director International Space University, Strasbourg)
- Tom Logsdon** (Senior member of Technical Staff, Space Division, Rockwell International)
- F. Landis Markley** (NASA, Goddard Space Flight Center)
- Robert G. Melton** (Professor of Aerospace Engineering, Pennsylvania State University)
- Keiken Ninomiya** (Professor, Institute of Space & Astronautical Science)
- Jehangir J. Pocha** (Letchworth, Herts.)
- Rex W. Ridenoure** (CEO and Co-founder at Ecliptic Enterprises Corporation)
- Gael Squibb** (Jet Propulsion Laboratory, California Institute of Technology)
- Martin Sweeting** (Professor of Satellite Engineering, University of Surrey)
- David A. Vallado** (Senior Research Astrodynamist, CSSI/AGI)
- Richard Van Allen** (Vice President and Director, Space Systems Division, Microcosm, Inc.)

More information about this series at <http://www.springer.com/series/6575>

James Miller

# Planetary Spacecraft Navigation

 Springer

James Miller  
Porter Ranch  
CA, USA

Space Technology Library  
ISBN 978-3-319-78915-6      ISBN 978-3-319-78916-3 (eBook)  
<https://doi.org/10.1007/978-3-319-78916-3>

Library of Congress Control Number: 2018945424

© Springer International Publishing AG, part of Springer Nature 2019

This work is subject to copyright. All rights are reserved by the Publisher, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilms or in any other physical way, and transmission or information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed.

The use of general descriptive names, registered names, trademarks, service marks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

The publisher, the authors and the editors are safe to assume that the advice and information in this book are believed to be true and accurate at the date of publication. Neither the publisher nor the authors or the editors give a warranty, express or implied, with respect to the material contained herein or for any errors or omissions that may have been made. The publisher remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

This Springer imprint is published by the registered company Springer Nature Switzerland AG  
The registered company address is: Gewerbestrasse 11, 6330 Cham, Switzerland

# Preface

This book is based on my 50 years of experience in navigation of Earth entry vehicles, Minuteman ballistic missiles, and planetary spacecraft. At the Jet Propulsion Laboratory, I worked on the Mariner 6, Viking, Pioneer, Galileo, and Near Earth Asteroid Rendezvous missions. At KinetX Inc. I worked on the MESSENGER and New Horizons missions. In writing this book I have drawn on engineering memoranda, conference papers, and publications I have written as well as many notes that I have accumulated over many years. My purpose is to present a book that will describe how navigation is done. The emphasis is on mathematics that have been coded in computer programs used for mission operations. Therefore, the derivations are given in detail since relatively little mathematics actually makes it into operational software. The mathematics that do are generally straightforward, but the programs are large and complex and must be virtually error free. In writing this book, I have frequently checked the mathematics by looking at computer code that I am confident is correct.

Chapter 1 contains the equations of motion exclusive of the force models. Since navigation is concerned with everything that moves, the equations of motion include the kinetic theory of gasses and propagation of electromagnetic waves. The force models are given in Chap. 2. Since motion is in a straight line without force being applied, the force models enable spacecraft to go somewhere. Chapter 3 describes the procedure for designing the trajectory a spacecraft will follow. A detailed derivation of Kepler's equation and Lambert's theorem is given. Just about all trajectory design is based on these two individuals' work aside from Newton of course. Trajectory optimization is described in Chap. 4. Most trajectory optimization is performed by developing an intuitive feel for the problem being solved. Sometimes it is necessary to perform a detailed constrained parameter optimization when there are many more control parameters than constraints. The first constrained parameter optimization using a computer program was the Viking orbit insertion maneuver. Previously, the trajectory designs were mostly Hohmann transfers. Chapter 5 describes the probability and statistics needed for navigation analysis. This is probably the most important navigation design function since it relates directly to the probability of achieving mission success. Orbit determination

is described in Chap. 6. At the beginning of planetary spacecraft navigation, orbit determination was a major problem. The Mariner class spacecraft had to contend with orbit determination errors of several thousand kilometers. Missions were flown so we could do orbit determination, not science. Today, the orbit determination error at Mars is on the order of tens of kilometers. This progress may be attributed to the introduction of VLBI. There is still the problem of orbit convergence and this problem is addressed in some detail. The measurements and calibrations are described in Chap. 7. Navigation is primarily concerned with the physical quantity being measured and not the hardware required to perform the measurement. However, some detailed knowledge of the measurement implementation is necessary to write navigation software. The navigation system is described in Chap. 8. The navigation system and navigation operations procedures are constantly evolving. An overview is given that applies to all navigation systems. It is conservatively estimated that there is at least one navigation system for each person doing navigation operations. The final chapter is anecdotal and describes some navigation analyses I performed over the years. My purpose is to describe the type of analyses the reader would be expected to perform if he or she pursued a career in navigation. I hope I have succeeded in convincing the reader that navigation is a lot of fun.

Planetary spacecraft navigation is the result of the work of many individuals including the author of this book. The person who had the original idea is often not known. I have mentioned some individuals in the text who I am aware of and made some significant contribution. Some are contemporary and known personally, but often these individuals are mathematicians and have lived over a 100 years ago. I have made little effort to search the literature and track down the original source. Most of my acknowledgments are anecdotal and the source is discussions in the coffee room that have not been verified. There are a few who have contributed directly to the writing of this book. My wife, Dr. Connie Weeks, Professor Emeritus of Mathematics, Loyola Marymount University, is the source of a large amount of my limited knowledge of mathematics. I cannot recall a question about mathematics where she did not have an immediate answer. The person most responsible for this book being written is Dr. Gerald Hintz who teaches at the University of Southern California. I have known him for about 50 years and since I started writing this book 20 years ago, he has kept encouraging me to finish. Writing this book has not been as difficult as I thought it would be, but rather an enjoyable experience. I must also acknowledge my mother, Eunice Miller, who thought it would be a good idea for me to be the first one on either side of my family to go to college. My sisters, Peggy Joyce, Nan Elizabeth, and Linda Lee, also got behind this major effort and I will be forever indebted to them.

I am particularly indebted to the editors at Springer International Publishing. Hannah Kaufman and Maury Solomon showed great patience in leading me through the process. The production people headed by Batmanadan Karthikeyan transformed my manuscript into what I regard as a work of art.

# Contents

<b>1</b>	<b>Equations of Motion</b> .....	1
1.1	Introduction .....	1
1.2	Particle Dynamics .....	1
1.3	n-Body Equations of Motion .....	4
1.4	Translational Variational Equations .....	6
1.5	Rotational Equations of Motion .....	7
1.6	Rotational Variational Equations .....	14
1.7	Kinetic Theory of Gases .....	18
1.8	General Relativity Equations of Motion .....	22
1.8.1	Einstein Field Equation .....	23
1.8.2	Geodesic Equation .....	24
1.8.3	Isotropic Schwarzschild Coordinates .....	27
1.8.4	Mercury Perihelion Shift .....	29
1.8.5	Radar Delay .....	30
1.8.6	Light Deflection .....	39
1.9	Numerical Integration .....	41
1.9.1	Fundamental Theorem of Calculus .....	41
1.9.2	Runge-Kutta Numerical Integration .....	43
	Bibliography .....	49
<b>2</b>	<b>Force Models</b> .....	51
2.1	Rocket Equation .....	51
2.2	Aerodynamic Forces .....	53
2.3	Solar Pressure .....	54
2.4	Gravity Models .....	56
2.4.1	Harmonic Expansion Model .....	57
2.4.2	Point Mass Model .....	58
2.4.3	Pyramid Model .....	59
2.4.4	Polyhedral Model .....	62
2.4.5	Mass Distribution of an Irregularly Shaped Body .....	63

- 2.4.6 Pyramid Gravity Model Comparison with Eros Harmonic Model ..... 66
- 2.4.7 Comparison of Gravity Model Mass Distributions ..... 67
- 2.4.8 Comparison of Density Distributions for Eros-Shaped Gravity Models ..... 69
- 2.4.9 Comparison of Gravity Model Accelerations ..... 71
- 2.4.10 Comparison of Gravity Model Accuracy and Computational Speed ..... 73
- 2.4.11 Gravitational Variational Equations ..... 75
- 2.5 Shape Model ..... 76
  - 2.5.1 Triangular Plate Model ..... 76
  - 2.5.2 Harmonic Expansion Shape Model ..... 77
  - 2.5.3 Gravity Harmonic Expansion from Shape Model ..... 79
- 2.6 Comet Atmosphere ..... 83
  - 2.6.1 Outgassing Model ..... 84
  - 2.6.2 Jet Model ..... 85
  - 2.6.3 Thermodynamic Model ..... 87
- 2.7 Summary ..... 91
- Bibliography ..... 92
- 3 Trajectory Design ..... 95**
  - 3.1 Restricted Two-Body Trajectories ..... 95
    - 3.1.1 Elliptical Orbit ..... 96
    - 3.1.2 Hyperbolic Orbit ..... 104
    - 3.1.3 Injection Flight Plane Hyperbolic Trajectory ..... 108
    - 3.1.4 Lambert’s Problem ..... 110
  - 3.2 Interplanetary Transfer ..... 116
    - 3.2.1 Hohmann Transfer ..... 117
  - 3.3 Three-Body Trajectory ..... 122
    - 3.3.1 Jacobi Integral ..... 122
    - 3.3.2 Tisserand’s Criterion ..... 124
    - 3.3.3 Gravity Assist Vector Diagram ..... 126
    - 3.3.4 Cassini Trajectory Design ..... 128
  - 3.4 Four-Body Trajectory ..... 131
    - 3.4.1 Moon Capture of Projectile Launched from Earth ..... 133
    - 3.4.2 Angular Momentum and Energy Management ..... 136
    - 3.4.3 Genesis Earth Return Trajectory ..... 139
    - 3.4.4 Jacobi Integral and Capture ..... 140
  - 3.5 NEAR Orbit Phase Trajectory Design ..... 143
    - 3.5.1 Spacecraft and Mission Constraints ..... 143
    - 3.5.2 Targeting Strategy ..... 144
    - 3.5.3 Targeting Algorithm ..... 145
    - 3.5.4 NEAR Trajectory Design ..... 147
    - 3.5.5 Approach Through 100 km Orbit ..... 148

3.5.6	Subsolar Overfly Through 50 km Orbit .....	149
3.5.7	Transfer to Southern Illuminated 35 km Orbit .....	150
3.5.8	Active POS Control, Polar Orbits, and Plane Flip .....	151
3.5.9	Northern Illuminated 50 km and 35 km Orbits .....	153
3.6	Summary .....	153
	Bibliography .....	155
<b>4</b>	<b>Trajectory Optimization .....</b>	<b>157</b>
4.1	Parameter Optimization .....	157
4.2	Statement of Problem .....	158
4.3	Condition for Optimum Solution .....	159
4.3.1	Lagrange Multipliers .....	161
4.3.2	Explicit Functions .....	162
4.3.3	Gradient Projection .....	163
4.4	Sample Problem .....	164
4.4.1	Solution by Method of Lagrange Multipliers .....	165
4.4.2	Solution by Method of Explicit Functions .....	165
4.4.3	Solution by Method of Gradient Projection .....	166
4.5	Second-Order Gradient Search .....	167
4.6	Inequality Constraints .....	169
4.7	Mission to Mercury .....	171
4.8	Multiple Encounter Optimization .....	174
4.8.1	Multiple Encounter Strategy .....	176
4.8.2	Trajectory Segment Optimization .....	179
4.8.3	Multiple Encounter Example .....	180
4.9	Summary .....	180
	Bibliography .....	184
<b>5</b>	<b>Probability and Statistics .....</b>	<b>187</b>
5.1	Normal Probability Distribution Function (PDF) .....	187
5.2	n-Dimensional Normal PDF .....	190
5.3	Bivariate Normal PDF .....	191
5.4	Rayleigh PDF .....	193
5.5	Central Limit Theorem .....	194
5.6	Monte Carlo Methods .....	195
5.7	Binomial Theorem .....	199
5.7.1	Confidence Limits .....	199
5.7.2	Normal PDF from Binomial Coefficients .....	201
5.7.3	Approximate Binomial Coefficients from Normal PDF .....	203
5.7.4	Stirling Approximation .....	204
5.8	Maxwell–Boltzmann Probability Distribution .....	207
5.8.1	Experimental Results .....	210
5.9	Summary .....	213
	Bibliography .....	215



<b>6</b>	<b>Orbit Determination</b> .....	217
6.1	Kalman Filter Algorithm .....	218
6.2	Weighted Least Squares .....	221
6.3	Square Root Information Filter (SRIF) .....	223
6.3.1	Discrete Process Noise Update .....	225
6.3.2	Solution Epoch .....	227
6.3.3	Computed and Consider Covariance .....	227
6.3.4	Smoothing .....	229
6.4	Continuous Filter Equations .....	230
6.4.1	Process Noise Term .....	233
6.4.2	Data Update Term .....	233
6.4.3	Continuous Filter Differential Equations .....	234
6.5	Continuous SRIF with Discrete Data Update .....	237
6.5.1	Process Noise Duality .....	237
6.5.2	Numerical Integration of SRIF Matrix .....	239
6.6	Direct Orbit Determination .....	240
6.6.1	Model of Doppler Data Signature .....	242
6.6.2	Parameterization of Doppler Signature .....	245
6.6.3	Solution by Newton-Raphson .....	247
6.6.4	Magellan Example .....	248
6.7	Summary .....	251
	Bibliography .....	253
<b>7</b>	<b>Measurements and Calibrations</b> .....	255
7.1	Radiometric Tracking Data .....	255
7.1.1	Doppler Data .....	256
7.1.2	Doppler Measurement Model .....	256
7.1.3	Data Noise .....	260
7.1.4	One-Way Doppler Data .....	261
7.1.5	Three-Way Doppler Data .....	262
7.1.6	Range Data .....	263
7.1.7	Very Long Baseline Interferometry .....	265
7.1.8	Differential Wide Band VLBI .....	266
7.1.9	Differential Narrow Band VLBI .....	267
7.2	Radiometric Data Calibrations .....	268
7.2.1	Clock Calibration .....	269
7.2.2	Troposphere Calibration .....	271
7.2.3	Ionosphere Calibration .....	272
7.2.4	Earth Platform .....	273
7.2.5	Polar Motion .....	273
7.2.6	Continental Drift .....	273
7.2.7	Solid Earth Tide .....	274
7.2.8	Plane Wave Propagation Through Ionized Gas .....	274
7.2.9	Solar Plasma Time Delay .....	277

- 7.3 Optical Data ..... 278
  - 7.3.1 Optical Data Processing ..... 279
  - 7.3.2 Planetary and Stellar Aberration ..... 280
- 7.4 Altimetry ..... 285
  - 7.4.1 Altimetry Data Measurement Model ..... 286
  - 7.4.2 Altimetry Variational Partial Derivatives ..... 289
- 7.5 Summary ..... 291
- Bibliography ..... 291
- 8 Navigation Operations ..... 293**
  - 8.1 Navigation System ..... 294
    - 8.1.1 Deep Space Network ..... 294
    - 8.1.2 Spacecraft ..... 294
  - 8.2 Orbit Determination ..... 295
    - 8.2.1 Orbit Determination Strategy ..... 296
    - 8.2.2 Multiple Data Types ..... 296
    - 8.2.3 Simulated Data ..... 297
  - 8.3 Maneuver Targeting ..... 298
    - 8.3.1 Interplanetary Maneuvers ..... 298
    - 8.3.2 In Orbit Maneuvers ..... 299
    - 8.3.3 K Matrix ..... 299
  - 8.4 Summary ..... 301
- 9 Navigation Analysis ..... 303**
  - 9.1 Viking ..... 304
    - 9.1.1 Planetary Quarantine ..... 304
    - 9.1.2 Orbit Insertion Maneuver Design ..... 306
  - 9.2 Galileo ..... 308
    - 9.2.1 Probe Delivery to Jupiter ..... 308
    - 9.2.2 Gravity Focusing ..... 309
    - 9.2.3 Probe Entry Dispersions ..... 310
    - 9.2.4 Probe Entry Flight Path Angle ..... 311
    - 9.2.5 Probe Entry Angle-of-Attack ..... 312
    - 9.2.6 Trajectory Bending ..... 313
    - 9.2.7 Jupiter Approach Orbit Determination ..... 314
    - 9.2.8 Relay Link ..... 315
    - 9.2.9 Jupiter Orbit Insertion ..... 317
    - 9.2.10 Probe Entry Trajectory Reconstruction ..... 318
  - 9.3 Pioneer ..... 319
    - 9.3.1 Orbit Determination Strategy ..... 320
    - 9.3.2 Orbit Determination Results from 1980 ..... 320
    - 9.3.3 Estimation of Drag ..... 323
    - 9.3.4 Relating Drag  $\Delta V$  to Period Change ..... 324
    - 9.3.5 Covariance Analysis Results ..... 328

- 9.4 Near Earth Asteroid Rendezvous ..... 329
  - 9.4.1 Orbit Determination Strategy ..... 331
  - 9.4.2 Eros **A Priori** Physical Model ..... 333
  - 9.4.3 Orbit Determination Solution ..... 333
  - 9.4.4 Eros Results ..... 336
  - 9.4.5 Shape Model ..... 337
  - 9.4.6 Gravity Harmonics ..... 338
  - 9.4.7 Polar Motion ..... 341
- 9.5 MESSENGER ..... 343
  - 9.5.1 Initial Post-Launch Orbit Determination ..... 344
  - 9.5.2 Estimated Accelerations from Assumed Water Vapor ..... 349
  - 9.5.3 Curve Fitting with Exponential Functions ..... 352
- 9.6 New Horizons ..... 355
  - 9.6.1 Pluto and Charon Approach ..... 356
  - 9.6.2 Pluto Approach Time-of-Flight Determination ..... 357
  - 9.6.3 Pluto and Charon Approach Covariance Analysis ..... 361
  - 9.6.4 Spacecraft Orbit Reconstruction ..... 365
- 9.7 Phobos ..... 366
  - 9.7.1 Phobos Inertial Properties ..... 367
  - 9.7.2 Phobos Gravity Field ..... 368
  - 9.7.3 Phobos Rotational Dynamics ..... 369
  - 9.7.4 Analytic Approximation of Forced Libration ..... 370
- 9.8 Summary ..... 375
- Bibliography ..... 377
  
- Answers to Selected Exercises ..... 379**
  
- Index ..... 385**